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EFFECT OF TAPERING ON THE PERFORMANCE OF WASTE STABILIZATION PONDS

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Abstract—A model for wastewater degradation in a tapered waste stabilization pond was derived as a modified Bessel function by materials balance approach. Based on hypothetical data, the tapered model gave lower faecal bacteria removal than the conventional (rectangular) model for various values of dispersion number, die-off rate coefficient, average width and shape factors. The above results were corroborated by data collected from two laboratory ponds operated in parallel; one having a tapered and the other a rectangular surface area. The latter gave slightly higher hydraulic efficiency and BOD_5 removal. Besides, faecal bacteria removal was significantly lower in the tapered pond than in the rectangular pond at 0.10 level of significance. Calculated faecal bacteria reduction using the tapered model was in good agreement with measured data with coefficient of correlation and standard error of 0.904 and 0.014, respectively. Effects of tapering on ponds with respect to construction cost, operational and maintenance ease and accuracy of estimated design parameters are also discussed. © 2001 Elsevier Science Ltd. All rights reserved

Key words—tapered waste stabilization ponds, shape effect, bacteria reduction, rectangular pond

NOMENCLATURE

a	coefficient accounting for the effects of d, k_0 and θ
A	pond surface area, m^2
A_1	cross-sectional area of upstreams side of elemental slice, m^2
b	outlet pond width, m
B	inlet pond width, m
$\bar{B}$	average pond width, m
B_1, B_2	arbitrary constants dependent on boundary condition
d	dispersion number
D	dispersion coefficient, $m^2\,day^{-1}$
h	pond depth, m
I_v (λm)	modified Bessel function, first kind
I_v^1 (λm)	derivative of k_v (λm) with respect to m
L	pond length, m
L_S	concentration of soluble BOD, mg/l
N	faecal coliform bacteria number, per 100 ml
N_e	effluent faecal coliform bacteria number, per 100 ml
N_o	influent faecal coliform bacteria number, per 100 ml
Q	flow rate, $m^3\,day^{-1}$
t	time, day
T	water temperature, °C
u	flow velocity, $m\,day^{-1}$
V	pond volume, m^3
y	variable pond width
Z	dimension less distance from pond surface to bed
	Greek symbols
Θ	detention time, days
λm	argument of the Bessel function
v	order of the Bessel function
δ	normalized variance of the t-concentration tracer curve
ϕ_1	function of I_v (λm)
ϕ_k	function of k_v (λm)
ψ	function of ϕ_1 and ϕ_k

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INTRODUCTION

A waste stabilization pond is a chemical reactor used for the reduction of solids as well as pathogenic organisms. It is a popular treatment option because of its high efficiency and low cost. Other advantages include: ability to absorb hydraulic and organic shock loads; tolerance of high concentration of heavy metals; low operating and maintenance cost and versatility in the treatment of wastewaters of different origins.

However, waste stabilization ponds are limited in application by their large area requirement (Mara *et al.*, 1983). In the past, researches have been conducted to improve pond efficiency thereby maximizing land use by optimization techniques (Agunwamba, 1991); using recirculating stabilization ponds in series (Shelef *et al.*, 1978); step feeding (Shelef *et al.*, 1987); incorporating an attached growth system (Shin and Polprasert, 1987); and more accurate estimation of pond design parameters (Polprasert *et al.*, 1983; Polprasert and Bhattarai, 1985; Sarikaya and Saatci, 1987; Sarikaya *et al.*, 1987; Marecos do Monte and Mara, 1987; Mayo, 1989; Agunwamba, 1992a, b; Agunwamba *et al.*, 1992). Besides, higher pond depths have been investigated for reduction of pond surface area (Hosetti and Patil, 1987; Oragui *et al.*, 1987; Silva *et al.*, 1987). However, no work seems to have been done on the utilization of ponds of different shapes to reduce land area.

It is conventional to design ponds of rectangular shape, but other shapes also need to be investigated. In most urban areas where good land is scarce, land for waste stabilization pond construction may be available only in certain shapes that may not allow for optimal land utilization if rectangular ponds are constructed. Hence, it is necessary to investigate whether other shapes that fit a given land site better will produce a higher quality effluent. The construction of ponds in such sites may not only reduce cost but also add to the aesthetic quality of the scenery.

Hence, in this research the performance of ponds with tapered surface areas is considered and compared with that of rectangular ponds of equal areas.

PROBLEM FORMULATION

The principle of conservation of mass is applied in order to obtain the dispersed flow equation for a waste stabilization pond with tapering sides.

Consider a surface area element of thickness Δx in the flow direction (x) and depth, h (Fig. 1(a)).

The widths at the upstream and downstream sections of the control volume are respectively y and $y-\Delta y$. All the symbols have been defined in Nomenclature section.

Mass inflow into the control volume in time interval Δt for advective flow is $QN\,\Delta t$ is and for dispersive flow it is $-DA_1(\partial N/\partial x)\Delta t$.

The corresponding quantities for outflow are $(QN+(Q\partial N/\partial x)\Delta x)$ and $-[DA_2\partial N/\partial x+(\partial/\partial x)(DA_2\partial N/\partial x)\Delta X]$, respectively.

The net mass of waste within the control volume is $A(\partial N/\partial t)\Delta x\,\Delta t$, whereas $-KNA\Delta t\,\Delta x$ is the reduction in amount due to degradation and K is the die-off rate coefficient.

Applying the general materials balance equation,

$$\begin{aligned}\text{Net Amount} &= \text{Inflow} - \text{Outflow}\\ &+ \text{Rate of Accumulation}\end{aligned} \tag{1}$$

the resultant equation becomes

$$\begin{aligned}A\frac{\partial N}{\partial t}\Delta x\Delta t &= \left(QN - DA_1\frac{\partial N}{\partial x}\right)\Delta t\\ &- \left(QN + \frac{Q\partial N}{\partial x}\Delta x\right)\Delta t\\ &+ \left[DA_2\frac{\partial N}{\partial x} + \frac{\partial}{\partial x}\left(DA_2\frac{\partial N}{\partial x}\right)\Delta x\right]\Delta t\\ &- KNA\Delta x\Delta t,\end{aligned} \tag{2}$$

If $A_1 = A_2 = A$, and $\partial N/\Delta t = 0$, equation (2) reduces to the form normally used in ponds for constant width (Polprasert *et al.*, 1983).

Knowledge of geometrical relationships between similar triangles (Fig. 1(b)) shows that

$$\frac{n}{b} = \frac{n+L-x}{y} = \frac{n+L}{B}. \tag{3}$$

By eliminating n,

$$y = \frac{BL - Bx + bx}{L} \tag{4}$$

Also, it is easily obtained from Fig. 1(b) that

$$\frac{\Delta x}{\Delta y} = \frac{L}{b-B}. \tag{5}$$

From Fig. 1(a) the values of A, A_1 and A_2 are, respectively, $(h/2)(2y-\Delta y)$, yh and $(y-\Delta y)h$. With these values substituted, equation (2) becomes

$$\begin{aligned}&-D\Delta y\frac{\partial N}{\partial x} - yu\frac{\partial N}{\partial x}\Delta x - \frac{1}{2}(2y-\Delta y)KN\Delta x\\ &+D(y-\Delta y)\Delta x\frac{\partial^2 N}{\partial x^2} + \frac{(b-B)}{L}D\frac{\partial N}{\partial x}\Delta x\\ &= \frac{1}{2}(2y-\Delta y)\frac{\partial N}{\partial t}\Delta x\end{aligned} \tag{6}$$

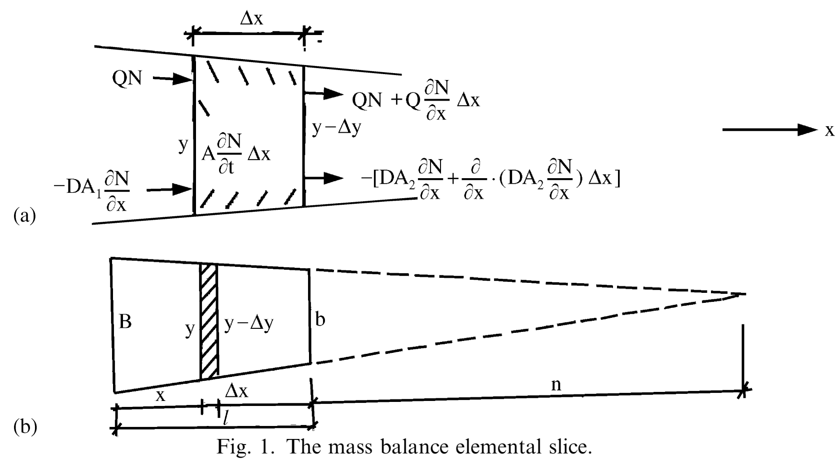

Fig. 1. The mass balance elemental slice.

If the terms with the product $\Delta x\,\Delta y$ are ignored, equation (6) becomes

$$-D\Delta y\frac{\partial N}{\partial x}-yu\frac{\partial N}{\partial x}\Delta x-yKN\Delta x$$
$$-\frac{(B-b)}{L}D\frac{\partial N}{\partial x}\Delta x+Dy\frac{\partial^2 N}{\partial x^2}\Delta x=y\frac{\partial N}{\partial t}\Delta x \tag{7}$$

Eliminating y and Δy from equation (7) with equations (4) and (5), respectively,

$$\frac{\partial N}{\partial t}=D\frac{\partial^2 N}{\partial x^2}-u\frac{\partial N}{\partial x}-\frac{D(B-b)}{BL-Bx+bx}\frac{\partial N}{\partial x}$$
$$-KN+\frac{(B-b)}{yL}D\frac{\partial N}{\partial x} \tag{8}$$

By continuity equation,

$$Q=yhu=\frac{hu}{L}(BL-Bx+bx) \tag{9}$$

i.e.

$$u=\frac{QL}{h(BL-Bx+bx)} \tag{10}$$

With u and y substituted for and $Z=x/L$, the steady-state form of equation (8) may be written as

$$\frac{d^2N}{dz^2}-\frac{QL}{hD(B-BZ+bZ)}\frac{dN}{dz}-\frac{KNL^2}{D}=0 \tag{11}$$

Further substitutions may be made to reduce equation (11) to the following standard form:

$$\frac{d^2N}{dm^2}+\frac{a_1}{ma_2}\frac{dN}{dm}-\frac{a_3N}{a_1^2}=0 \tag{12}$$

in which

$$a_1=\frac{QL}{hD}=\frac{Q}{hud}=\frac{\bar{B}}{d} \tag{13a}$$

where $d(=D/ul)$ is the dispersion number and $\bar{B}$ is the average width;

$$a_2=B-b \tag{13b}$$

$$a_3=\frac{KL^2}{D}=\frac{KL}{ud}=\frac{K\theta}{d} \tag{13c}$$

and

$$m=B-a_2Z \tag{13d}$$

where $\Theta(=L/u)$ is the detention time.

Equation (12) is a Bessel equation of the form

$$w''+\frac{(2v-1)w'}{S}-\lambda^2w=0 \tag{14}$$

The solution chosen depends on the problem being solved and whether or not v is an integer (Mclachlan, 1954). The complete solution whether or not v is an integer is (Mclachlan, 1954):

$$w=S^v[B_1I_v(S\lambda)+B_2K_v(S\lambda)] \tag{15}$$

where v and $S\lambda$ are the order and argument of the equation, respectively; I_v and K_v are the first and third kind of modified Bessel functions; and B_1 and B_2 are arbitrary constants to be determined by the boundary conditions. Hence, in terms of the variables used in this research the solution is

$$N=m^v[B_1I_v(\lambda m)+B_2K_v(\lambda m)], \tag{16}$$

in which

$$v=\frac{1}{2}\left(1-\frac{a_1}{a_2}\right),\qquad \lambda=\frac{1}{a_2}\sqrt{a_3} \tag{17}$$

BOUNDARY CONDITIONS

For mathematical consistency, Danckwerts boundary conditions are used as given below (Wehner and Wilhelm, 1956; Agunwamba, 1990):

$$N_o=N-d\frac{dN}{dm},\qquad z=0\quad\text{or}\quad m=B \tag{18}$$

$$\frac{dN}{dm}=0,\qquad z=1\quad\text{or}\quad m=b. \tag{19}$$

SOLUTION FOR PARTICULAR CASES

(i) Rectangular ponds: If $B=b$ then $a_2=0$ and equation (11) reduces to the normal differential equation for rectangular ponds with the solution (Wehner and Wilhelm, 1956)

$$\frac{N_e}{N_o}=\frac{4a^2}{(1+a)^2}\exp\left(\frac{1-a}{2d}\right) \tag{20}$$

where N_e is the effluent waste concentration and a is expressed as $a^2=1+4k\Theta d$.

(ii) Tapering ponds: If $B>b$ and $b>0$, then $m=B-(B-b)z$. At $z=0$, $m=B$. If $z=1$, $m=b$. Hence, $b\le m\le B$ for all values of z and so k (Bessel function) has no singularity for this condition. Noting that $(d/dm)I_v(\lambda m)=\lambda I_v'(\lambda m)$ and using the relationship (Mclachlain, 1954) $m\lambda I_v'(\lambda m)=vI_v(\lambda m)+\lambda mI_{v+1}(\lambda m)$ and $\lambda mK_v'(\lambda m)=vK_v(\lambda m)-\lambda mK_{v+1}(\lambda m)$, after differentiating equation (16) and substituting in equation (18), the first boundary condition gives

$$\frac{N_o}{B^{v-1}}=B_1I_v(\lambda B)[\lambda B-d\lambda v(b-B)+dv]$$
$$+B_1\,d\lambda BI_{v+1}(\lambda B)+B_2K_v(\lambda B)[\lambda B$$
$$-d\lambda v(b-B)+dv]-B_2d\lambda BK_{v+1}(\lambda B) \tag{21}$$

Similarly, the second boundary condition gives

$$B_1[\lambda v(b-B)I_v(\lambda b)+vI_v(\lambda b)+\lambda bI_{v+1}(\lambda b)]$$
$$=B_2[\lambda bK_{v+1}(\lambda b)-vK_v(\lambda b)-\lambda v(b-B)K_v(\lambda b)] \tag{22}$$

The values of B_1 and B_2 are obtained from these two equations and then substituted in equation (16).

The concentration at any distance downstream is therefore:

$$\frac{N}{N_o} = \frac{m^v}{B^{v-1}}\left[\frac{\phi_k}{\phi_I} I_v(\lambda m) + K_y(\lambda m)\right]\frac{1}{\varphi} \tag{23}$$

in which

$$\phi_1 = \lambda v(b-B)I_v(\lambda b) + vI_v(\lambda b) + vbI_{v+1}(\lambda b) \tag{24}$$

$$\phi_k = \lambda bK_{v+1}(\lambda b) - vk_v(\lambda b) - \lambda v(b-B)K_v(\lambda b) \tag{25}$$

and

$$\varphi = (\phi_k/\phi_I)\{I_v(\lambda B)[\lambda B - \mathrm{d}\lambda v(b-B) + \mathrm{d}v] + \mathrm{d}\lambda BI_{v+1}(\lambda B)\} + K_v(\lambda B)\{\lambda B - \mathrm{d}\lambda v(b - B) + \mathrm{d}v\} - \mathrm{d}\lambda BK_{v+1}(\lambda B). \tag{26}$$

At the effluent end $m = b$ and $N = N_e$.

Hence,

$$\frac{N_e}{N_o} = \frac{b^v}{B^{v-1}\varphi}\left[\frac{\phi_K}{\phi_I} I_v(\lambda b) + k_v(\lambda b)\right] \tag{27}$$

Using the Wronskian relationship (Abramowitz and Stegun, 1964), $I_v(\lambda b)K >_{v+1}(\lambda b) + I_{v+1}(\lambda b)K_v(\lambda b) = 1/\lambda b$, to simplify the numerator, equation (27) reduces to

$$\frac{N_e}{N_o} = \frac{b^v}{B^{v-1}\phi_I} \tag{28}$$

(iii) Triangular ponds: If $b = 0$, $m = B(1 - z)$ and specifically at $z = 1$, $m = 0$. Hence, $K_v(\lambda m)$ becomes infinite and ceases to be a solution

$$N = B_1 I_v(\lambda m)m^v, \tag{29}$$

With the first boundary condition, $B_1 = N_o/\{I_v(\lambda B)[B\lambda + dv\lambda B + vd] + \mathrm{d}BI_{v+1}(\lambda B)\}B^{v-1}$.

Therefore,

$$N = \frac{m^v N_o I_v(\lambda m)}{B^{v-1}\{I_v(\lambda B)[B\lambda + \mathrm{d}v\lambda B + v\mathrm{d}] + \mathrm{d}BI_{v+1}(\lambda B)\}} \tag{30}$$

The value of equation (3) is zero for $m = 0$ provided $v \neq 0$. A triangular pond is not feasible because b cannot be zero; otherwise there will be no flow.

(iv) Limiting case: This holds if a_1 is very large. If $a_1 \to \infty$ either $L \to \infty$ or $D \to 0$. Hence, $v \to \infty$ implies that equation (28) approximates plug flow.

THEORETICAL COMPARISON BETWEEN RECTANGULAR AND TAPERED PONDS

Effect of d on N_e/N_o

Consider a tapered pond with the following Parameters: $Q = 100\,\mathrm{m^3/day}$, $L = 100$ m, $B = 10$ m, $b = 7.5$ m, $h = 1.0$ m, $k = 0.5$/day. Therefore, the average rectangular pond width that will give equal area ($875\,\mathrm{m^2}$) with the tapered pond is 8.75 m.

With the above data, a_1, a_2, a_3 and hence λ and v for given values of d (0.2, 0.4, 0.6, 0.8, 1.0, 1.2) are obtained using equations (13) and (17).

The values of λ and v obtained enabled the evaluation of N_e/N_o using mathematical tables (Abramowitz and Stegun, 1964) and equation (28). The corresponding values of N_e/N_o for a rectangular pond are calculated from equation (20). This whole calculation is repeated for $Q = 191$ and $373\,\mathrm{m^3\,day^{-1}}$ and in all cases N_e/N_o values are obtained (Table 1). The results for both tapered and rectangular ponds are compared in Fig. 2. As expected, the higher the values of d the higher the bacteria reduction ratio (N_e/N_o) and hence the smaller the efficiency ($1 - N_e/N_o$) for both tapered and rectangular ponds. However, the efficiency of the tapered pond appears to be lower than that of the rectangular pond of equal surface areas.

Table 1. Effect of variation of dispersion number on faecal coliform bacteria reduction (N_e/N_o) ($B = 10$ m, $b = 7.5$ m, $h = 1$ m, $L = 100$ m, $k = 0.5\,\mathrm{day^{-1}}$)

Q $\mathrm{m^3\,day^{-1}}$	Θ days	d	λb	λB	Predicted N_e/N_o	
					Tapered pond	Rectangular pond
		0.2	14.0	18.7	0.077	0.053
		0.4	9.9	13.2	0.154	0.079
100	8.75	0.6	8.1	10.8	0.207	0.096
		0.8	7.0	9.4	0.248	0.108
		1.0	6.3	8.4	0.309	0.118
		1.2	5.7	7.6	0.359	0.125
		1.2	10.2	13.5	0.230	0.170
		0.4	7.2	9.6	0.327	0.203
191	4.58	0.6	5.9	7.8	0.406	0.222
		0.8	5.1	6.8	0.449	0.235
		1.0	4.5	6.1	0.464	0.244
		1.2	4.1	5.5	0.547	0.249
		0.2	7.3	9.7	0.384	0.264
		0.4	5.1	6.9	0.480	0.391
373	2.35	0.6	4.2	5.6	0.565	0.404
		0.8	3.6	4.9	0.586	0.412
		1.0	3.3	4.3	0.745	0.416
		1.2	3.0	4.0	0.740	0.418

Figure 2 seems to suggest the existence of greater disparity between the performances of the two ponds at higher dispersion numbers than at lower values. The slopes of the lines for the tapered pond are higher than those for the rectangular pond. For instance, at $d = 0.6$, the slopes of the lines for the tapered pond are 0.217, 0.333 and 0.233 for $Q = 100$, 191 and 373 $m^3\,day^{-1}$, respectively while the corresponding values for the rectangular ponds are equal to 0.05, 0.067 and 0.075, respectively. Hence, a tapered pond would respond faster to changes in dispersion than the corresponding rectangular pond. This is disadvantageous for the tapered pond since the effluent quality may be less stable. At very low d values, N_e/N_o predicted by the two models approach the same values.

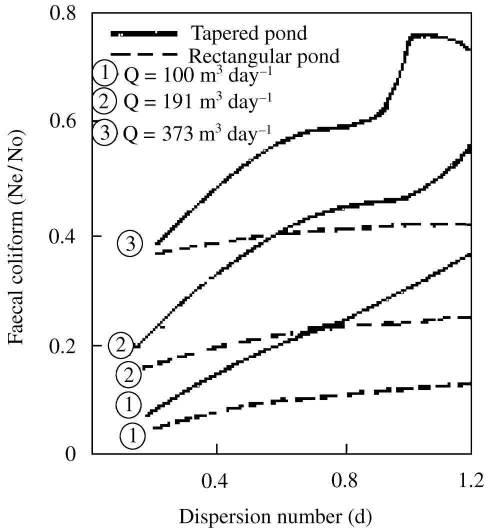

Fig. 2. Effect of d on N_e/N_o for different flow rates ($B = 10$ m, $b = 7.5$ m, $h = 1$ m, $L = 100$ m, $k = 0.5\,day^{-1}$).

Effect of average width $\bar{B}$ on N_e/N_o

The effect of variations of $\bar{B}$ (from 15 to 25.0) on N_e/N_o for different detention times ($\Theta = 1.50$, 3.75, 5.00, 7.5 and 10.00 days) are shown in Table 2 and Fig. 3 for the pond volume: V and B kept constant at 1500 m^3 and 25 m, respectively. The other parameters d, h, and k were equal to 0.25, 1 m and 0.5 day^{-1} respectively. As $B - b \rightarrow 0$, equation (8) reduces to that for rectangular ponds. In other words, for constant B as $\bar{B}$ increase, N_e/N_o approaches the value obtained using equation (20). At low detention time, because of the associated high velocity which increases rapidly down the converging pond, N_e/N_o fluctuates in value.

Table 2. Effect of variation of average width ($\bar{B}$) on faecal coliform bacteria reduction (N_e/N_o) ($B = 25$ m, $h = 1$ m, $V = 1500\,m^3$, $d = 0.250$, $k = 0.5\,day^{-1}$)

Θ days	$\bar{B}$ M	$b\lambda$	$B\lambda$	Predicted N_e/N_o
				Tapered pond
	15.0	0.5	2.3	0.631
	17.5	1.2	3.0	0.478
1.50	20.0	2.6	4.3	0.506
	22.5	7.0	8.8	0.447
				Rectangular pond
	25.0			0.514
				Tapered pond
	15.0	0.7	3.5	0.314
	17.5	1.8	4.5	0.277
3.75	20.0	4.1	6.8	0.264
	22.5	11.0	13.8	0.226
				Rectangular pond
	25.0			0.232
				Tapered pond
	15.0	0.8	4.0	0.236
	17.5	2.1	5.3	0.197
5.00	20.0	4.8	8.0	0.178
	22.5	12.6	15.8	0.171
				Rectangular pond
	25.0			0.159
				Tapered pond
	15.0	1.0	4.8	0.152
	17.5	2.6	6.5	0.139
7.5	20.0	5.9	9.8	0.115
	22.5	15.6	19.5	0.098
				Rectangular pond
	25.0			0.082
				Tapered pond
	15.0	1.1	5.5	0.198
	17.5	3.0	7.5	0.066
10.0	20.0	6.8	11.3	0.059
	22.5	17.8	22.3	0.052
				Rectangular pond
	25.0			0.045

Effect of shape factor (b/B) on N_e/N_o

The area and average width are kept constant at $875\,m^2$ and 8.75 m, respectively. The inlet width (B) and outlet width (b) are varied such that $\frac{1}{2}(B+b)=8.75\,m$. Then with $d=0.318$, $Q=100\,m^3\,day^{-1}$, $k=0.5\,day^{-1}$ and the length and depth being the values used previously, a_1, a_2 and a_3 and hence λ and v are computed. The values of N_e/N_o are finally obtained, and then the above calculations repeated for $Q=191$ and $373\,m^3\,day^{-1}$ as shown in Table 3. Fig. 4 shows the graphs of N_e/N_o against b/B for various values of Q. The sensitivity of N_e/N_o to changes in b/B is very low. As the shape factor increases by 300% N_e/N_o decreases by only 3.2, 4.7 and 3.2% for $Q=100$, 191 and $373\,m^3\,day^{-1}$, respectively. In all cases, the model for rectangular ponds gave the lowest values of N_e/N_o.

Effect of K on N_e/N_o

The values of K are varied from 0.1. to 1.0 and each time the corresponding values of a_3 computed while a_1 and a_2 are kept constant at 27.52 and 2.5, respectively (Table 4). The effect of k on N_e/N_o is shown in Fig. 5 for $v=-5$, $B=10\,m$, $b=7.5\,m$, $d=0.318$, $h=1\,m$ and $L=100\,m$. N_e/N_o decreases as the die-off rate coefficient increases for both ponds. The rectangular pond is less sensitive to variations in k than the tapered pond for the three Q values and gave lower bacteria reduction ratios (higher efficiencies).

EXPERIMENTAL METHODS

The effects of shape on BOD_5 removal and bacteria reduction were investigated on a laboratory-scale ponds with a rectangular and a tapered shape (on plan) in the Public Health Engineering Laboratory of the University of

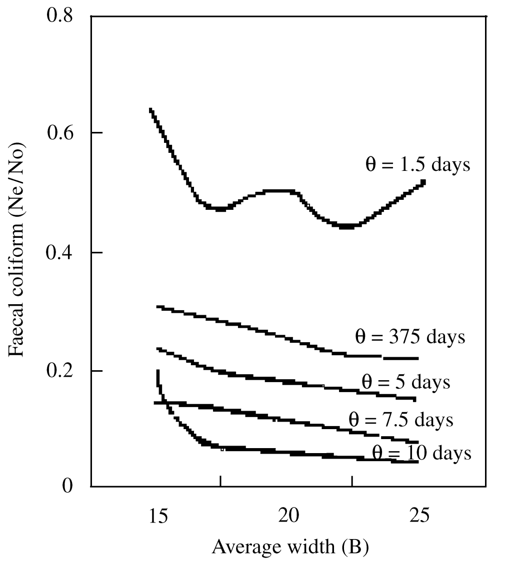

Fig. 3. Effect of average $\bar{B}$ on N_e/N_o for different Θ ($B=25\,m$, $h=1\,m$, $V=1500\,m^3$, $d=0.25$, $K=0.5\,day^{-1}$).

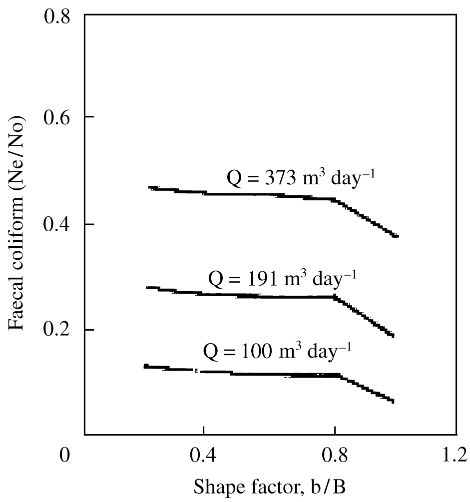

Fig. 4. Effect of shape factor on N_e/N_o for different flow rates ($A=875\,m^3$, $L=100\,m$, $h=1\,m$, $d=0.318$, $K=0.5\,day^{-1}$).

Table 3. Effect of variation of shape factor (b/B) on faecal coliform bacteria reduction (N_e/N_o) ($A=875\,m^2$, $L=100\,m$, $h=1\,m$, $d=0.318\,m$, $k=0.5\,day^{-1}$)

$Q\,m^3\,day^{-1}$	$\frac{b}{B}$	$b\lambda$	$B\lambda$	Predicted N_e/N_o
	0.2	0.9	4.6	0.125
	0.4	2.5	6.2	0.122
100	0.6	5.6	9.3	0.121
	0.8	14.9	18.6	0.121
	1.0	*(Rectangular pond)*		0.069
	0.2	0.7	3.4	0.278
	0.4	1.8	4.5	0.273
191	0.6	4.0	6.7	0.265
	0.8	10.8	13.5	0.265
	1.0	*(Rectangular pond)*		0.192
	0.2	0.5	2.4	0.468
	0.4	1.3	3.2	0.456
373	0.6	2.9	4.8	0.455
	0.8	7.7	9.6	0.453
	1.0	*(Rectangular pond)*		0.382

Table 4. Effect of variation of faecal coliform die-off rate constant (K) on faecal coliform bacteria reduction (N_e/N_o) ($B = 10$ m, $b = 7.5$ m, $h = 1$ m, $L = 100$ m, d = 0.318)

Q m^3 day^{-1}	K	$b\lambda$	$B\lambda$	Predicted N_e/N_o	
				Tapered pond	Rectangular pond
	0.1	5.0	6.6	0.625	0.473
	0.2	7.0	9.4	0.345	0.263
100	0.4	10.0	13.3	0.176	0.103
	0.6	12.2	16.3	0.088	0.048
	0.8	14.1	18.8	0.052	0.025
	1.0	15.7	21.0	0.030	0.104
	0.1	3.6	4.8	0.743	0.658
	0.2	5.1	6.8	0.566	0.459
	0.4	7.2	9.6	0.354	0.250
191	0.6	8.8	11.8	0.219	0.150
	0.8	10.2	13.6	0.161	0.095
	1.0	11.4	15.2	0.110	0.063
	0.1	2.6	3.4	0.957	0.799
	0.2	3.7	4.9	0.763	0.651
	0.4	5.2	4.9	0.568	0.451
373	0.6	6.3	8.4	0.437	0.326
	0.8	7.3	9.7	0.358	0.243
	1.0	8.2	10.9	0.283	0.185

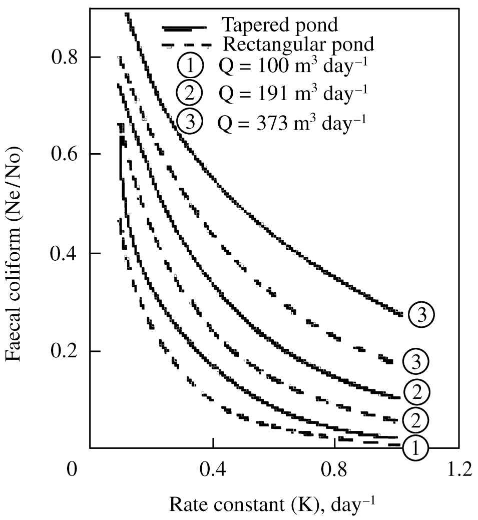

Fig. 5. Effect of k on N_e/N_o for different flow rates ($B = 10$ m, $b = 7.5$ m, $h = 1$ m, $L = 100$ m, $d = 0.318$).

Nigeria, Nsukka. The length, width and depth of the rectangular-shaped pond are 1.0, 0.4 and 0.18 m, respectively. The larger and smaller width of the tapered pond are 0.6 and 0.2 m, respectively with a depth of 0.18 m. Two PVC tanks (200 and 150 l were used as reservoirs of domestic wastewater collected from the first facultative pond of the treatment plant at the University of Nigeria, Nsukka. The 150 l PVC tank served to maintain the volume of the second reservoir at a constant flow. The two were connected with a 19 mm diameter pipe. The other details are as indicated in Fig. 6.

Tracer studies

These were conducted to determine the dispersion characteristics of the two ponds for different detention times. Sodium chloride (NaCl) was used as the impulse tracer and the response tracer concentration was monitored at the exit stream at fixed time intervals. The amount of input impulse tracer was about 60 g for each pond. The base amount of NaCl present in the wastewater was subtracted from the measured values. The calculation of the dispersion number (d) was made with the relationship (Levenspiel, 1962; Marecos do Monte and Mara, 1987)

$$\sigma^2 = 2d - 2d^2\left[1 - \exp\left(-\frac{1}{d}\right)\right], \tag{31}$$

where σ^2 is the normalized variance determined from the tracer concentration–time curves.

Physicochemical analysis

Procedures described in Standard Methods (APHA, 1985) were used to determine the parameters. Samples were collected once in a week and analyzed for pH, dissolved oxygen (DO), influent and effluent five-days biochemical oxygen demand (BOD_5) and water temperature. DO and pH were measured by DO meter (TOA Co) and pH meter, respectively. Pond temperatures were obtained with a thermometer.

Bacteriological analysis

The faecal coliform removal efficiency of the ponds was determined by collecting grab samples of the influent and the effluent once per week. The most probable number (MPN) of faecal coliform method was used.

Flow measurement

The wastewater discharge was measured using a graduated discharge measurement tank because no flow meter and recorder were available. The discharge was obtained from the recorded volume divided by the time to reach that volume. The flow in the tapered pond was adjusted with the control valves to achieve the same flow rate in the rectangular pond.

The two ponds were continuously fed with waste water from the reservoir for two weeks to attain steady-state condition (Shin and Polprasert, 1987).

Evaluation of other model coefficients

From the experimental results reported in Table 5, the faecal coliform die-off rate coefficient (k) was determined

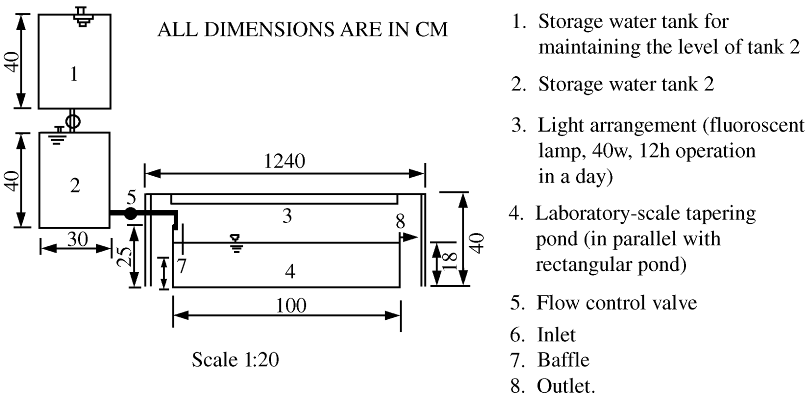

Fig. 6. Schematic profile of laboratory scale tapering pond. (1) Storage water tank for maintaining the level of tank 2. (2) Storage water tank 2. (3) Light arrangement (fluorescent lamp, 40 W, 12 h operation in a day). (4) Laboratory-scale tapering pond (in parallel with rectangular pond). (5) Flow control valve. (6) Inlet. (7) Baffle. (8) Outlet.

Table 5. Operational characteristics of the tapered rectangular ponds

Expt. No.	Detention times (Θ days)	pH	Temperature (T°C)	DO (Mg/L)	Influent MPN $\times 10^6$ 100 ml^{-1}	Influent BOD$_5$ Ls (mg/L)	K (day^{-1})
1	6.9	8.0	25.0	7.4	9	258.0	0.753
2	7.7	7.8	25.5	7.8	23	214.0	0.683
3	5.9	8.0	25.0	5.8	39	192.3	0.788
4	11.0	7.6	24.0	7.1	23	182.6	0.645
5	6.5	7.5	24.0	6.1	9	222.8	0.561
6	8.4	7.5	23.5	7.2	43	222.3	0.561
7	8.5	8.1	26.5	7.4	43	217.0	0.891
8	6.6	7.5	24.0	7.9	15	253.8	0.528
9	10.1	7.6	24.5	7.4	9	206.3	0.618
10	7.4	8.1	25.0	7.1	23	225.0	0.776
11	9.5	8.0	25.0	7.0	9	214.8	0.707
12	6.4	8.1	25.0	7.2	14	172.0	0.776
Mean	7.9	7.8	24.8	6.5	21.6	215.4	0.702
S.D.	1.6	0.3	0.8	2.1	13.4	25.5	0.1084

with the formula (Saqqar and Pescod, 1992)

$$k = 0.5(1.02)^{T-20}(1.15)^{(\mathrm{pH}-6)2}(0.99784)^{(L_3-100)} \quad (32)$$

where T and L_s are the water temperature and filtered BOD$_5$, respectively.

As before, the effluent bacteria reduction ratio for the rectangular and tapered ponds were evaluated using equations (20) and (28), respectively, based on the dimensions of the tapered pond: $B = 0.6$ and $b = 0.2$. These values together with the reported values of d, k and Θ enabled the determination of a_1, a_2, and a_3; then v and λ and subsequently, N_e/N_o for the tapered pond. The calculated and measured N_e/N_o as well as BOD$_5$ removal efficiency and dispersion numbers in both ponds are reported in Table 6.

EXPERIMENTAL RESULTS AND DISCUSSION

Experimental comparison between the two ponds

Hydraulic efficiency. The value of the dispersion number is a measure of the hydraulic efficiency of a waste stabilization pond. Table 6 shows the d values for both tapered and rectangular ponds. The average d value in the tapered pond was 0.127 against the value of 0.118 for the rectangular pond. In the former case, the surface boundaries are converging. The velocities vary with location and increases as the channel becomes narrower (Weber, 1978). Hence, dispersion in the tapered pond is subject to higher variation. This is supported by the corresponding values of coefficient of variation, 63.8 and 46.6% in the tapered and rectangular ponds, respectively (Fig. 7). The effect of the tapering was a slight reduction in the pond's hydraulic efficiency, and hence in bacteria removal efficiency. At low dispersion, the flow is subjected to conditions of low turbulence. Short-circuiting is minimized and the two ponds approach plug flow.

Removal efficiency. Table 6 also shows the data for bacteria and BOD$_5$ removal. The average N_e/N_o for the tapered pond is higher, implying lower efficiency. Following the normal small sample theory of test of hypothesis (Spiegel, 1987), the student t-critical value (at 22 degrees of freedom and 0.10 level of significance) is 1.32 while the computed t value is 1.39. Hence, at this level it is significant to infer that N_e/N_o for the tapered pond is greater than that of

Table 6. BOD_5 removal (%), faecal coliform bacteria reduction and dispersion number in the tapered and rectangular ponds

Expt. No.	Tapered pond					Rectangular pond			
	d	BOD removal (%)	Measured d (N_e/N_o)	Calculated		d	BOD removal (%)	Measured d (N_e/N_o)	Calculated (N_e/N_o)
				Tapered model	Rectangular model				
1	0.049	61.9	0.044	0.014	0.013	0.049	70.0	0.021	0.013
2	0.091	57.3	0.026	0.039	0.019	0.091	77.9	0.027	0.019
3	0.079	60.6	0.030	0.036	0.026	0.090	55.1	0.020	0.028
4	0.101	60.1	0.011	0.013	0.008	0.122	74.2	0.017	0.009
5	0.072	70.2	0.052	0.044	0.048	0.101	76.2	0.060	0.056
6	0.285	68.2	0.096	0.095	0.056	0.088	55.0	0.048	0.026
7	0.056	74.0	0.004	0.007	0.003	0.058	62.7	0.009	0.003
8	0.284	53.9	0.159	0.100	0.099	0.101	63.1	0.060	0.062
9	0.068	73.9	0.012	0.044	0.008	0.250	74.0	0.050	0.027
10	0.162	62.1	0.039	0.039	0.023	0.162	69.2	0.016	0.023
11	0.137	72.6	0.020	0.016	0.012	0.158	72.0	0.021	0.014
12	0.137	58.8	0.091	0.085	0.030	0.146	61.6	0.033	0.032
$\bar{X}$	0.127	64.5	0.047	0.046	0.029	0.118	67.6	0.032	0.026
S.D.	0.081	7.0	0.031	0.047	0.027	0.055	7.9	0.018	0.018

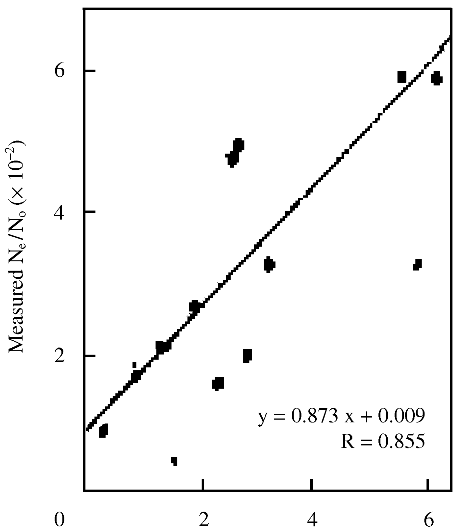

Fig. 7. Measured versus calculated in rectangular pond.

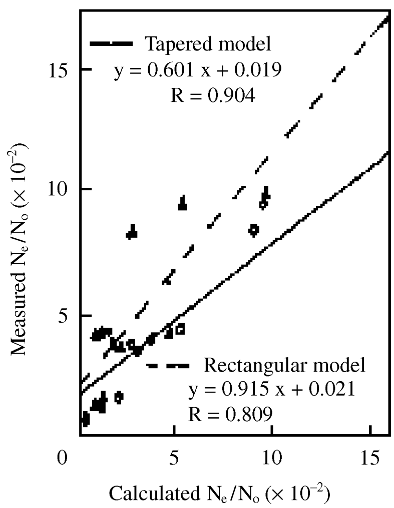

Fig. 8. Measured versus calculated N_e/N_o in tapered pond (○ = tapered model. ▲ = rectangular model).

the rectangular pond. Probably, the significance will be higher at higher d values if other factors (k, Θ, and pond dimensions) are kept constant (Fig. 2).

The average BOD_5 removal was very slightly higher in the tapered pond than in the rectangular pond. However, statistically the difference is not significant at 0.10 level of significance.

Verification of models. The verification of the conventional model is shown in Fig. 8. There is a good correlation ($R = 0.855$) between the measured and calculated N_e/N_o with a low standard error (0.010). Similar good correlation has been obtained by previous authors (see e.g. Polprasert *et al.*, 1983). As for the tapered pond, the coefficient of correlation and standard error when the measured N_e/N_o was regressed on the calculated N_e/N_o are 0.904 and 0.014, respectively. If instead of equation (28), equation (20) (rectangular model) is used for the prediction, the corresponding values are 0.809 and 0.021, respectively. The R values for the tapered model is significantly higher than that of the rectangular model at 0.10 level of significance.

The ponds are further compared with respect to ease of maintenance, construction cost, and ease of computation.

The simple maintenance process in rectangular ponds include cutting the grass around the pond edges; removal of scum on the pond surface, and debris and coarse material removal at the pond inlet and outlet. These activities seem not to be affected by a pond's shape.

With respect to construction cost, the setting out of a rectangular pond is easier than that of a tapered pond.

The estimation of the design parameters like flow velocity and dispersion number are more difficult because of the shape effect. Flow velocity varies from point to point along the length of the pond and the design value may be taken as the average over the whole pond. The value of d is even more difficult to estimate since it is influenced by pond shape as well as other complicated factors that depend on shape (e.g. friction factor) (Agunwamba *et al.*, 1992; Agunwamba, 1992a, b).

The computation of N_e/N_o in a tapered pond is greatly simplified by already available mathematical tables (Abramowitz and Stegun, 1964). Despite this the computations are still more involved than for a rectangular pond.

CONCLUSION

A dispersed flow model based on materials balance was derived for tapered waste stabilization ponds, Equation (28) was proposed for determination of bacteria reduction in tapered ponds. With hypothetical data, higher bacteria removal efficiency was obtained for the conventional (rectangular) model than for the tapered model for different values of dispersion number, average width, shape factor and die-off rate coefficient. Similarly, experimental results revealed higher faecal bacteria removal performance, BOD_5 removal and hydraulic efficiency for the rectangular pond. However, calculated bacteria reduction with the tapered model was in good agreement with laboratory data collected from a tapered pond. If at all tapered ponds can find applications in congested areas where land is scarce or unavailable in a shape that permits the optimal construction of rectangular ponds, such advantage has to be balanced against the lowered efficiency, higher computational efforts and, probably, higher construction cost.

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