FMTL_x L_y L_z DIMENSIONAL EQUATION FOR SLUDGE DRYING BED.

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TO

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CHAPTER ONE

1.01 INTRODUCTION

Waste generation, a growing problem facing the world is as a result of rapid industrialization, urbanization and increase in population growth. Emphasis on agriculture for more food production to take care of the expansion in population has generated increased volume of domestic, industrial as well as agricultural wastes.

Owing to health and other environmental hazards (like water, air and soil pollution) industrial and domestic wastewaters pose to the existence of man and the entire ecosystem there is therefore, every need to treat wastewater properly before discharging to receiving streams, water bodies or agricultural lands.

Sludge, a term used to designate the solid that settles when sewage is passed through a settling tank is a serious problem for all wastewater treatment plants all over the world in terms of disposal and handling. Industrial as well as domestic wastewater processing in an effort to meet more stringent discharge measures and limitations led to increased sludge production.

Sludge which is produced as a by-product of all treatment processes have considerable application potentials (such as fertilizer and soil conditioners). The liquid sludge contains about 90-98% water and sometimes it is transported long distance for ultimate disposal. Economically, it is very important to reduce the sludge volume by removing as much water as possible to ease the transportation, handling and disposal cost.

Dewatering and disposal of sludge is a major economical factor in the operation of wastewater treatment plant. MIKKELSEN, (2002) reported that 30-
50% of the annual operating costs of treatment plants are related to sludge dewatering alone.

Sludge dewatering can be done through a natural means- drying bed. Drying beds are mostly used in small industries as well as small community with a population of over 100,000 people, (Echenfelder and Santhanam, 1981). The assumptions in designing drying beds are mainly based on the solid content as well as the sludge volume. Sludge drying beds are often open to the air and the performance or efficiency is dependent on weather and climatic conditions. It equally depends on the nature of the sludge, the height of the sludge applied on the drying bed and this is usually between 150mm- 350mm and the dry initial solid content of the sludge. Solar intensity, wind velocity and temperature also play important role in the efficient performance of drying beds though they are often neglected. Moisture reduction on the drying bed is lost through percolation and evaporation.

Dispersed particles in sludge had often been observed to have a negative effects on filtration rate and the increase in resistance experienced using drying bed is due to the blinding of the filter medium of sludge cake with small particle. (Mehrdadi, et al,2006).

FMTL (Force, Mass, Time and Length) dimensional analysis is a mathematical technique used in checking derived equations according to Rajput, (1998). It is used in solving several engineering problems. Since no chemical reaction is assumed to take place during sludge filtration, dimensional analysis can be used in deriving an equation to describe the system.
1.02 Research Aims and Objectives.

(1) To use dimensional analysis approach of FMTL_xL_yL_z to develop sludge filtration equation for sand drying bed without the compressibility attribute.

(2) To examine the effect of chemical conditioning on specific resistance using sand drying bed.

(3) To demonstrate or investigate the performance of sand drying bed as sludge dewatering option.

1.03 Research Problems.

Since mechanical method of dewatering sludge is very expensive in terms of operation, as well as equipment and maintenance cost. The research work is to develop a dewatering option that will be more economical to maintain and operate.

1.04 Significance of Study.

To maintain a healthy environment is a goal worthwhile, and hence the great need to treat properly our waste water before they find their way into our water bodies or agricultural lands and other possible reuse. Over the years mechanical method of dewatering have proved very expensive both in maintenance and cost of equipment. As a result, researchers in this field are concerned with developing a dewatering technique like the natural drying bed to reduce the high maintenance and equipment costs of dewatering sludge.

This work highlights the use of dimensional analysis approach to derive sludge filtration equation which can be found useful in the design and operation of sand drying beds.
1.05 Scope and Limitation of the Research.

The scope is limited to the treatment of sewage sludge from the waste water treatment plant at the University Of Nigeria, Nsukka. Owing to the weather and climatic condition at the time the research was carried out, sludge treatment was adversely affected as longer time was needed to attain the optimum temperature needed to intensify dewatering.
CHAPTER TWO

2.01 LITERATURE REVIEW.

Sludge disposal is a growing challenge all over the world as most waste water treatment plants in an effort to meet or adhere to the recommended discharge limits lead to more waste generation.

Reduction in sludge volume by dewatering before drying and subsequent disposal is very important; as the water content of the sludge is reduced it enhances the cost of handling, disposal as well as transportation cost. Most often than not, to utilize this sludge for different applications such as fertilizer and soil conditioners it is sometimes transported over a long distance.

Economically, sludge volume reduction will significantly reduce the cost of transportation, barging trips and time (in a situation where sludge is to be deposited in a river, sea or ocean.) handling and disposal of the waste.

Ademiluyi, et al, (1983), reported that because of the high water content in sludge which is about 97.5%, sludge generated is difficult to handle.

According to Agunwamba, (2001), dewatering process increases solid contents of the sludge between 20-35%. In the same way, Grag (.2008) agreed that dewatering of sewage sludge does not only remove the high water content, but also produce sludge that is non putrescible and odourless.

2.02 Sludge Treatment Processes.

Sludge treatment processes are aimed at achieving a low sludge mass for disposal i.e a low solid mass and a high dry matter content of the dewatered sludge cake, and destruction or stabilization of sludge solids. While digestion,
accomplish solid destruction and stabilization, sludge volume reduction is achieved by both concentration and dewatering. Both volume reduction and solid destruction are also possible via heat drying and combustion. (Ademiluyi, J.O. 1981).

Several techniques exist to dewater sludge. Most of them are mechanically done using a filter press (employing pressure or vacuum) or a centrifuge. The other methods are based on natural phenomenon such as evaporation, percolation, temperature, wind velocity, precipitation or rainfall to achieve sludge volume reduction. These include the natural drying bed, lagoons, oxidation ponds, bio-filters, oxidation ditch etc. All these technologies especially drying beds and oxidation ponds require large areas of land and with the current increase in industrialization and urbanisation to acquire such area of land is a scarce resource.

2.03 Sludge Digestion

Sludge digestion is a decomposition process which can either be aerobically (in the presence of oxygen) or anaerobically (absent of oxygen). Anaerobic digestion is usually mesophylic at a temperature of 35°C. The major thing in sludge digestion is that living organisms eat up or consume the organic matters in the sludge and the fermentation process that result from the sludge decomposition reduces the pollution that comes from the sludge.

This process is accompanied by liquefaction, production of gas (methane gas) and stabilization of organic matter. The gas so produced can be useful in producing heat or electrical energy. The digested sludge can be used as soil conditioner for agricultural purposes and can also be incinerated. (Fair, et al, 1971) developed an equation, a relationship for sludge digestion as,
Bacteria + Organic matter = Bacteria + CH\textsubscript{4} + CO\textsubscript{2} + H\textsubscript{2}O + Residue.

2.04 Anaerobic Digestion

The purpose of sludge digestion is to convert bulky odorous raw sludge to a relatively inert material that can be rapidly dewatered with the absence of obnoxious odour. The bacterial processes consist of two successive processes that occur simultaneously in digesting sludge. The first stage consists of breaking down large organic compounds and converting them to organic acids along with gaseous by-products of carbon dioxide, methane and trace amount of hydrogen sulphide. This step is performed by a variety of facultative bacteria operating in an environment devoid of oxygen. If the process were to stop there, the accumulated acids would lower the pH and would inhibit further decomposition by PICKLING the remaining raw wastes. In order for digestion to occur, second-stage gasification is needed to convert the organic acids to methane-forming bacteria which are strict anaerobes and are very sensitive to environmental conditions of temperature, pH and anaerobiosis. In addition, methane bacteria have a slower growth rate than the acid former and are very specific in food supply requirements. (Ademiluyi, J.O. 1981).

2.05 Thickening

Raw sludge has about 97.5% water content and 2.5% solid and thickening is a treatment process used in increasing the solid content of the sludge by reducing the water content of the sludge. This reduction process is important since it enables the engineer or designer to predict the capacity of the tank or the equipment requirements of other treatment units.
2.06  Sludge Conditioning

The conditioning of waste sludge involves pre-treatment, this helps to facilitate water removal during subsequent thickening or dewatering operation. During conditioning, small and amorphous particles are transformed into larger and stronger aggregates. This process increases the rate of water drainage and solid separation. In most dewatering process the ability of sludge to form and maintain a porous media that enhances its compressibility is the desired goal (Clarke et al. 1997).

Sludge conditioning process may be achieved through physical, biological or chemical means. The chemical conditioning process involves the addition of coagulants which is usually organic or inorganic compounds such as ferric chloride or aluminum sulphate. While the physical conditioning involves heating the sludge to a very high temperature or by freezing the sludge which cause the formation of ice crystals that grows by the addition of water molecules to their structures. The solid particles can then be easily separated from the sludge. Also the use of synthetic polymers for sludge conditioning is adopted, example the Moringa oleifera seeds. (Suleyman Aremu Muyibi, et al.2001).

2.07  Dewatering.

This wastewater treatment process increases solid content of the sludge between 20 to 35% and as a result, the sludge output can be handled and transported easily. This makes disposal economically cheap as it reduces the barging time and the number of trips required. To achieve this, conditioning of the sludge using coagulants (ferric chloride, fecl₃ etc.) is usually required. Sludge may be dewatered through natural drying and this include the use of drying beds, flash
drying or thermal drying, etc or by mechanical dewatering options which include centrifuges, filter presses, vacuum filters etc. (Ademiluyi, J.O. 1981).

2.08 Centrifugation.

Centrifuges are employed for dewatering raw, digested, and waste activated sludge. Typical operation produces a cake with 15-30% solids concentration depending on the character of the sludge. In the absence of chemical conditioning the solids capture is in the range of 80%, while proper chemical pre treatment can increase solids recovery to 90%. (Ademiluyi, J.O. 1981).

2.09 Flash Drying.

Flash drying is the term applied to a process for drying the sludge particles in suspension in a stream of hot gases which provide practically instantaneous removal of the moisture by evaporation. This technique is usually employed where there is cheap source of energy. (Ernest, W. Steel.1960).

2.10 Incineration and Drying.

Sludge drying involves reducing water content by vaporizing the water to air. Incineration is an extension of the drying process and it converts solids into an inert ash that can be disposed of easily. Some modern incinerators are equipped with energy recovery devices which enable energy to be recycled back for incineration of subsequent waste or utilized as electrical energy.

Other heat treating processes applied in sludge incinerations are flash-drying incineration system and the fluidized bed reactor. Flash drying involves pulverizing wet sludge cake with recycled dried solids in a cage mill. Hot gases from the incinerating furnace suspend the dispersed sludge particle up into a
pipe duct where they are dried. However, a cyclone separator removes the dried solids from the moisture-laden hot gas, which is returned to the furnace. At this point part of the dried sludge returns to the mixer for blending with the incoming wet cake. The remainder is either withdrawn for fertilizer or incinerated.

A fluidized bed reactor uses a sand bed that is kept in fluid condition by an up flow of air, as a heat reservoir to promote uniform combustion of sludge solids. The bed is pre-heated to approximately 650°C by using fuel oil or gas. When sludge cake is introduced, rapid combustion occurs maintaining an operating temperature of about 800°C. Ash and water vapour are carried out with the combustion gases. Cyclonic wet scrubber is used to remove ash from the exhaust gases. (Metcalf and Eddy, 2004).

2.11 Drying Bed.

Natural drying process is performed by the use of drying beds. In this case, dewatering process is achieved by loosing water to the atmosphere by evaporation and also through percolation through the filter medium via the under drain. Normally, the filtrate that results as a result of drainage through the filter medium is returned to the treatment plant for subsequent treatment. Drying beds are more economical to operate and maintain when compared to mechanical means, though large area of land is often required , it is often used in small industries or communities for waste treatment.

Sludge drying time depends on weather and climatic condition, the thickness of the sludge applied, and also the nature of the sludge. It may vary from 10 days to several weeks. Addition of coagulant such as the traditional ferric chloride or Aluminum sulphate expedites dewatering. To increase the speed and reduce
interference from weather, sludge drying bed may be covered with structures similar to those used for green houses effect. (Ernest, W. Steel.1960).

2.12 Typical Drying Bed.

A drying bed consist of a 300mm sand layer under lain by fine and coarse graded gravel, that envelope tile or perforated pipe under drains. Sludge is applied on the sand bed and is allowed to dry by evaporation and drainage of excess water over a period of several weeks depending on weather or climatic conditions and some other factors like the nature and thickness of the sludge applied etc. Decomposition of the sludge by bacterial action takes place during the drying process at sufficiently high moisture content. Large beds are partitioned by concrete walls into sections 8m wide by 30-60m long. A pipe header, with gated opening to each cell, is used to apply digested sludge to the bed. Seepage or filtrate collected in the under drains is returned to the treatment plant wet well. The total area of open drying beds at a treatment plant is usually 0.1 to 0.2m²/BOD design population equivalent. Digested sludge applied to a depth of 200-250mm dries to a fibrous layer of 100mm in a period of few weeks. With sufficient bed area and digester storage volume, this process of dewatering can be compatible for most climatic regions. It is worthy to note that during rainy season the process may take a longer time frame to complete and as a matter of fact sizing the area of the drying beds should take this into account. However, the laborious process of removing the digested cake is a major problem for operating personnel. Although some plants have mechanical equipment, the time-honoured method is manual removal using a shovel like fork. Attempting to employ front-end loaders leads to disturbance of the bed and excessive damage and loss of sand. (Metcalf and Eddy, 2004).
2.13 Design of Drying Bed.

To design a drying bed, two methods are basically utilized, the empirical method and the rational method. To calculate empirically, the area is computed as the product of the population and the loading rate. Typical loading rate for different kinds of sludges are as shown below;

Loading Rates of Sludge on Drying Beds.

<table>
<thead>
<tr>
<th>S/N</th>
<th>Sludge loading rate (m²/s)</th>
<th>Open beds</th>
<th>Covered beds</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>Primary digested</td>
<td>0.30 – 0.45</td>
<td>0.20 – 0.30</td>
</tr>
<tr>
<td>2</td>
<td>Primary &amp; humans digested</td>
<td>0.40 – 0.50</td>
<td>0.30 – 0.40</td>
</tr>
<tr>
<td>3</td>
<td>Primary &amp; activated digested</td>
<td>0.50 – 0.75</td>
<td>0.40 – 0.45</td>
</tr>
<tr>
<td>4</td>
<td>Primary &amp; chemically precipitated digested</td>
<td>0.60 – 0.75</td>
<td>0.40 – 0.45</td>
</tr>
</tbody>
</table>

Source: (Agunwamba, 2001).

On the other hand, the rational equation for the design of drying bed is dependent on time for sludge to drain ($t_1$) and time ($t_2$) for moisture to evaporate from the drained sludge. Swanwick, et al, (1961) expressed the total time as,

$$ T = t_1 + t_2 \text{ } \text{ } \text{ } \text{ } (2.0a) $$

Where $T$ = total residence time of the sludge on the bed.

$t_1$ = time taken for sludge to drain.

And $t_2 = \frac{30HS_0}{aE - bR} \left( \frac{1}{S_1} - \frac{1}{S_2} \right) \text{ } \text{ } \text{ } \text{ } (2.0b)$$

Where $H$ = Initial sludge application rate.
\[ R = \text{Rainfall in wet month.} \]
\[ E = \text{Clear water evaporation rate (mm month).} \]
\[ a = \text{Correction factor for evaporation from sludge.} \]
\[ T = \text{Total residence time of the sludge on the bed in a typical wet month.} \]
\[ S_o = \text{Initial percent solids.} \]
\[ S_1 = \text{Percent solids after t, days.} \]
\[ S_2 = \text{Desired final percent solids.} \]

The surface area (A) for an assumed depth (H) is obtained from the relationship,

\[
A = \frac{Q}{H}^T \]

Since water evaporates slower from sludge than clear water, rainfall also tend to increase the moisture content, as a result, \( aE - bR \) is the effective evaporation. The value 30 in equation (2.0 b) above is a conversion factor from months to days. Typical values of a, and b are, 0.75 and 0.57 respectively.

### 2.14 Types of Drying Bed.

(1) Conventional sand dying beds.
(2) Artificial media drying beds.
(3) Paved drying beds.
(4) Solar drying beds.
(5) Vacuum–assisted drying beds.
2.15 Paved Drying Bed.

Most often these type of beds are rectangular in shape and measures about 6-15 meters wide and 21-45 meters long. Asphalt, concrete as well as cement are used as liner to prevent seepage in these beds. It is easy to maintain and permits access to equipments. This type of bed can be used in an alternative to the sand drying beds. It has drainage type and a decanting type. The paved decanting drying bed is advantageous in warm, semi-arid and arid climates.

Drainage type works in similar way with the conventional sand drying beds as the under drain is collected and sludge removal is enhanced by the use of a front end loader.

2.16 Vacuum - Assisted Drying Bed.

This drying bed method helps in increasing sludge volume reduction and drying. It makes use of vacuum pump to accelerate dewatering process and drying. Filtrate percolates through the multimedia filter into a sump. Sludge is applied to a depth between 300mm and 750mm. Conditioners are injected through the inlet line to improve the performance of the filtration process. During the operation of the bed a pumping system is started, after the sludge has been drained by gravity, the process continues until the cake cracks and vacuum is lost, this eventually is followed by solid removal. (Metcalf and Eddy, 2004).

2.17 Solar Drying Bed.

This bed type utilizes energy from the sun to achieve dewatering operation and drying. It makes use of sophisticated technology like that of GREEN HOUSE.
2.18 Design Criteria.

Generally, the size of the drying bed must be in accordance with the data gotten from a similar treatment plant within the same geographical location and has the same influent characteristics.

The wall of drying beds must be water tight and should extend to 300mm to 600mm above and should not be less than 150mm below the surface of the bed. The bottom of a drying bed must have at least 300mm layer of clayey subsoil of permeability between 10-7 centimetre per second (cm/sec.). Where the ground water table is within 1.2 metres an impermeable concrete pad must be installed over a liner.

Sand Media Beds: should have at least 300mm sand bed with coefficient of uniformity of less than 4.0 and effective grain size of not less than 0.3mm. The sand above the top of the under drain should not be above 75mm thick.

Gravel Media Bed: The top layer of gravel media bed must be at least 75mm thick and of size between 3.1mm and 6.3mm. The thickness of the gravel around the under drain should be about 300mm and should be graded properly in size. A gravel media bed must be laid in two or more layers.

Open Beds: The drying bed must be sizable enough to store accumulated or excess sludge during unfavourable weather conditions such as extended rainfall and high humidity. Sludge dewatering must be effective and must have at least two beds. There must be room also for effective dewatering during high and extended rainfall, such as that provided for GREEN HOUSE.

The efficacy of natural drying bed is dependent on some factors.
(1) Nature of sludge: Raw sludge and especially those containing high amount of oil or grease, tend to dry slowly by evaporation above a dry solids content of about 30%, while digested sludge crack faster forming a highly fragmented cake which in favourable weather or climate, will produce a dry solid content as high as 70%. Sludge which drains readily leaves a cake of relatively uniform dry solid content and the one which drains slowly produces a cake which is wet and stick to the lower layer of the filter medium.

(2) Weather and climatic condition: Water evaporates slower from sludge than clean water. Sludge dewatering is also at the mercy of rainfall as moisture reduction through evaporation is minimum in cloudy conditions due to reduced intensity from the sun. A well digested sludge 200-300mm thick can dry within one week or two without odour under favourable climatic condition.

(3) Dry initial solid content of sludge: It has often been observed that dispersed particles has a negative effect on rate of filtration, therefore it is better to consolidate or thicken the sludge before application so as to reduce the proportion of liquor which was to be removed through percolation or drainage.

(4) Height of sludge applied: This is usually between 150-350mm thick with mechanically lift beds. The depth of application is often 200mm. If the applied depth is too shallow, the thickness of the sludge layer will be small and more applications will be required to deal with a given volume of sludge.

Solar intensity, wind velocity and temperature also affect the performance of drying bed though they are often neglected.

Sludge drying bed as method of sludge treatment has some advantages as well as disadvantages. (Mehrdadi, N. , et al.2006).
2.19 Disadvantages.

(1) Sludge drying bed is open to the public and constitute grate environmental hazard to man and the entire ecosystem due to its septic and unpleasant odour.
(2) Large area of land is required.
(3) Sludge drying beds (open beds) are seriously affected during drying process by weather and climatic change.
(4) It is labour intensive in removing the dry cake as vehicles are not allowed to stand or drive through the sand dry bed.

2.20 Advantages of Drying Beds.

(1) Operation of sludge drying beds requires no skills, little or no attention is needed.
(2) It is economical to operate and maintain, when compared to other dewatering methods.
(3) The operation results in consolidated cake after drying process.
2.21 SPECIFIC RESISTANCE

This is an outstanding parameter in sludge filtration process. It is used in quantifying the filterability of sludge. It was observed that during the removal of solid from the parent liquids which is referred to as sludge filtration, the solid particles that are usually suspended in the prefilt are deposited at the surface of the filter septum while the liquid content passes through Ademiluyi, (1986). As the process is in progress, particles are deposited and are referred to as filter cake.

Consequently, there is increase in the thickness of the cake deposited and for non rigid cake, the increase in thickness causes compression till the cake cracks. Ademiluyi, (1986), showed that the major force needed to overcome the resistance offered by the cake and the filter septum is provided by the difference in pressure between the deposition zone of the cake and the bottom of the filter medium. The resistance offered to the filtrate can therefore determine the dewaterability or filterability of the sludge. However, the lower the value of the specific resistance the more filterable the sludge.

Dilution and chemical conditioning of sludge has also shown significant effect on the specific resistance and compressibility coefficient of sludge.

Carman (1938), who first proposed the idea of specific resistance on rigid cake filtration showed that specific resistance \( r \), is defined as the pressure required to procure unit rate flow of liquid of unit viscosity through unit cube of the cake. Assuming the flow obeys Darcy’s law, for a rigid cake of thickness \( L \), the rate of filtration is then given by,

\[
\frac{dv}{dt} = \frac{PA}{r \mu L} \quad \text{and} \quad r = \frac{1}{k_1} \quad \text{.......................... 2.1.}
\]
Where, \( P \) = pressure, \( A \) = Area of filtration, \( L \) = Thickness of cake, \( R \) = Specific resistance, \( \mu \) = Viscosity, \( V \) = Volume of filtrate, \( t \) = Time of filtration, \( K_1 \) = Permeability coefficient.

In equation 2.1, the specific resistance \( r \), is used instead of the permeability \( K_1 \).

Over the years, researchers in the field have been reporting specific resistance \( R \), in different units and this has caused disparity and confusion and equally the loss of bench mark. However, the various units of specific resistance lack fundamental reasons.

Christensen, (1983), came out with a modification in the unit of specific resistance value and this is based on the adoption of the SI system of units. He advocated the use of appropriate multiples like, \( m/kg \) to avoid the use of complicated scientific notations. He suggested a numerical value between 0.1 and 1000 and eventually choose tetrameter per kilogram \( (Tm/kg) \). For outstanding higher or lower specific resistance values, Christensen suggested that petametres per kilogram \( (Pm/kg) \) or gigametres per kilogram \( (Gm/kg) \) respectively may be preferred.

Ademiluyi, (1986), noted that specific resistance does not have consistent units though an outstanding parameter in measuring sludge filterability. Specific resistance depends on sludge type, the pressure applied and the condition of measurement.

Ademiluyi, and Egbuniwe, (1984), reported that the lower the value of specific resistance the more dewaterable or filterable the sludge is.
2.22 THEORY OF FILTRATION.

Sludge filtration is a process which involves the separation of the solid component from the parent liquid through a porous medium or septum. The solid particles in the sludge are deposited on the surface of the filter medium or septum where they continue to build up forming an increasing thicker cake. Along the line, the cake so formed provide most of the filtration action, and as more solids are deposited there is a corresponding increase in pressure as well as the resistance across the cake thickness. For non rigid cake, it will continue to compress until the cake cracks.

Haas, et al, (1995), showed that cake so formed does not change its volume as pressure builds up. Also for non rigid cake, the pressure across the cake increases faster than the cake build up.

Almy and Lewis, (1912), demonstrated filtration process by filtering chromium hydroxide in a small plate and frame press at unchanged pressure. From their investigation, they proposed that the filtrate flow rate was a power function of pressure as well as volume of filtrate. They suggested equation of the form:

\[
\frac{dv}{dt} = \frac{kp^n}{vm} \nonumber \tag{2.2} 
\]

Which they agreed constituted the basic law of filtration, n and m are indefinite powers associated with pressure p and volume v respectively. K is a proportionality constant and it varies with the material to be filtered as well as the condition of operation.

On the bases of the similarity between filtration process and ground water flow, Sperry, (1916), suggested another filtration equation. He opined that in as much
as Poiseuille’s law agrees with or holds for ground water flow, it should also be adopted as basic law of filtration. Though his suggestion was based on theoretical rather than experimental point of view, he derived a general equation in which rate of flow was shown to be proportional to the first power of \( p \) and \( v \), equally room was given for the effect of resistance on the filter-base and filtrate viscosity variation. He modified Poiseuille’s law as,

\[
\frac{dv}{dt} = \frac{p}{R} \quad \text{.................................................. (2.3)}
\]

Where \( p = \text{pressure} \), \( R = \text{the resistance to the flow of filtrate through the filter cake and the supporting septum} \).

The above equations suggested by Almy and Lewis as well as that suggested by Sperry have been proved invalid both on experimental and theoretical ground by many researchers in the field (Ruth, 1935; Heertjes, 1964; Anazodo, 1974; and Halff, 1952).

Weber and Hershey, (1926) did not agree with the original equation proposed by Almy and Lewis but modified it into this form,

\[
\frac{dv}{dt} = \frac{pA^{2+\theta}}{r_p \int (dv)^\theta (dt) d(\nu')} = \frac{p^{1-s}A^{2+\theta}}{r'' v \int (dv)^\theta (dt) dv} \quad \text{........................... (2.4)}
\]

For the general case, the integration form of the above equation was given as;

\[
\frac{(V)^{2+\theta}}{(A)} = \frac{p^{1-s}}{r''(1+\theta)v} \left(\frac{(2+\theta)^{1+\theta}}{(1+\theta^t)}\right) \quad \text{........................... (2.5)}
\]

Since Poiseuille’s law and D’Arcy’s law both originated from the permeability of liquid through bed of porous material and considering sand bed as a bundle of capillaries; Carman(1934b,1938) expressed D’Arcy’s law as,
\[ U = \frac{k_1 p}{\mu L} \] .......................... (2.6)

Where \( k_1 \) = the permeability coefficient.

Assuming the total filtration pressure \( p \), is made up of the part which overcomes the resistance of cake and another which overcomes the initial resistance of the filter septum, then for non compressible cake of thickness \( L \) and assuming that filter septum also obeys D’Arcy’s law (which shows that flow through sand bed is directly proportional to the pressure difference and inversely proportional to the length of flow), Carman expressed the rate of filtration as,

\[ U = \frac{1}{A} \frac{dv}{dt} = \frac{k_1 p}{\mu L} \] .......................... (2.7)

Where \( A \) = Area of filtration, \( p \) = pressure drop across the cake, \( L \) = cake thickness.

Cake permeability is defined as the ease with which liquid pass through the filter septum while cake resistance is defined as the difficulty with which liquid is passed, assuming that cake thickness;

\[ L = \frac{cV}{A} \]

Carman expressed the equation of filtration as,

\[ \frac{dv}{dt} = \frac{PA^2}{\mu (rcV + AR)} \] .......................... (2.8)

Where \( R \), is the initial resistance due to the filter medium or septum and \( r \) is the resistance of the cake.

Integrating equation (2.8) at constant pressure
\[ t = \frac{\mu r c V}{2 P A^2} + \frac{\mu R V}{P A} \]

Implying that,

\[ \frac{t}{V} = \frac{\mu r c V}{2 P A^2} + \frac{\mu R}{P A} \] \hspace{1cm} (2.9).

Equation (2.9), is considered a typical straight line equation and when \( \frac{t}{V} \) is plotted against \( V \) it gives a slope,

\[ b = \frac{\mu r c}{2 P A^2} \] \hspace{1cm} (2.10)

The specific resistance of cake is given as,

\[ r = \frac{2 b P A^2}{\mu c} \]

Even though equation (2.9) originated from rigid cake concept, Carman suggested that it could be used for compressible cake as well. Carman assumed \( r = r^1 P^s \) \hspace{1cm} (2.11)

Where \( s \) in equation (2.11) is widely accepted as compressibility coefficient of sludge cake. For rigid cake, \( s \) is taken as zero, that is \( s = 0 \)

Carman’s work on filtration was based on the idea of specific resistance and the time velocity plot at constant pressure. He assumed that specific resistance is independent of the suspended solid concentration and opined that the total loss of pressure arises from pressure drop across filter cake, pressure drop across initial resistance and loss incurred in filtrate recovery. However Carman’s work has generated a lot of criticism from other investigators in the field since it is based on ideal situation where the cake that results during filtration process is considered rigid at constant pressure.
Ruth, (1935) showed that Carman’s equation does not give room for low compressibility values to be evaluated with any degree of accuracy and for a rigid cake, the compressibility coefficient \( S_c = 0 \) and agreed that for low compressibility, \( S_c \) should be equal to unity. He equally postulated that compressibility coefficient is a measure of cake compressibility and formulated an equation of the form,

\[
    r = r_1 (1 + r_2 S_c) \quad \text{.........................(2.12)}
\]

Where \( r_1 \) and \( r_2 \) = Constants, \( r \) = specific resistance.

Hemant, (1981) argued that Ruth’s filtration theory was incomplete since it could not explain most of the filtration data at constant pressure, he showed that the theory did not take into consideration particle migration within a cake. On this note he modified Ruth’s theory by adding empirically particle migration in the cake for cases involving dilute slurries.

The modified equation by Hemant is given as,

\[
    \log V = \frac{1}{2+a} \log t + \frac{1}{2+a} \log \frac{(2+a)p}{\mu a v c} \quad \text{..................(2.13)}
\]

On the variability of specific resistance, Hermant claimed that the assumption that average specific resistance is constant is not true for analysis of data collected only about 20 minutes.

He equally opined that in the constant pressure filtration tests, the flow rate continuously decreases with time and so does the pressure \( P_m \) across the medium. He did not account for the variability of pressure in his derivation but only considered it constant. In Hermant’s derivation, it implies that pressure in
(constant) pressure filtration can only be assumed constant only when the hydrostatic head is zero.

In 1953, Tiller demonstrated theoretically that when volume of filtration $v$ is plotted against time $t$ at constant pressure that the curves so formed were not perfect parabolas. He showed that if at any time the pressure drop across the filter medium were a reasonable fraction of the pressure loss across the cake, the average filtration resistance was not constant and the plot of $t/v$ against $t$ curves were not straight. On this note, the assumption that the rate of flow as well as the average porosity that were always assumed constant and independent of distance through the filter bed were found to be invalid.

Tiller, opined that in ordinary filtration investigation, the solids closest to the filter medium are closely packed than at any other portion of the cake. As a result the porosity is minimum at the point of contact between the cake and the filter medium and maximum at the point where the liquid enters. Owing to the fact that in filtration process the resistance depends on the porosity of the filter medium, the resistance increases steadily as the filtrate passes through the cake assuming the solids are compressible. Sjenitzer in 1955, reported that rate of flow is inversely proportional to cake thickness and this implies filtration resistance is directly proportional to the thickness of the cake. On the variability of resistance, $r$ during any filtration cycle, Gale (1967), suggested that one would expect it because in as much as the filtration progress the depth of the cake increases and so also does its resistance to flow.

Grace in 1953, theoretically showed that the specific resistance $r_p$ for a cake with uniformly applied pressure stress, could be obtained by consolidating it at that pressure and later determine its permeability coefficient. He suggested
an expression for the average specific resistance of the cake which depends on the pressure drop across the cake as $P_2 - P_1$ and on the pressure drop across the filter medium as $P_1 - P$. The septum resistance is determined from separate permeability experiment as; $P_2 - P_1 = (P_2 - P_0)$ .................(2.14a.).

$$\int_0^{P_2 - P_1} \frac{dp}{rp} = \frac{P_2 - P_1}{r} \quad \ldots \ldots \ldots \ldots \ldots \ldots \ldots (2.14b).$$

By integrating the left hand side of the equation using Simpson’s rule the value of $r$ is obtained. Grace, pointed out that the average specific resistance at any particular pressure within the required range could be calculated and that the determination of permeability over a complete range of pressure could be achieved in a day. He claimed that in the filtration of compressible cakes the mechanical pressure on the cake particles varies through the cake depth, causing a variation in the cake porosity and specific resistance through the depth of the deposited cake. Grace, opined that the average specific cake resistance would necessarily be expressed in terms of average cake porosity. Grace, also asserted that the pressure that is causing physical compression of the cake result in the cumulative drag of filtrate flowing through cake, and kinetic energy change in filtrate flowing through the cake. Since the overall pressure stress increases in the direction of filtrate flow, the specific cake resistance increases in the same direction.

Perry, and Dobson, (1971) reported that gas bubble formation within the growing filter cake is a significant factor in controlling of filtration process. Other researchers, Shriato, et al in 1972, pointed out that cake filtration theory constitute variable internal flow rates. They validated this theory by considering experimental variation of porosity and hydraulic pressure within the cake.
this was in agreement with the result that showed experimentally that porosity decreases with increasing pressure.

Coakley, (1956), modified the filtration equation suggested by Carman (1934b, 1938) he showed that it is difficult to measure the mass per unit volume of cake for compressible sludge. He suggested that the rate of filtration should be written thus,

\[
\frac{dv}{dt} = \frac{PA^2}{\mu (\alpha c + R_m A)}
\]

.......................... (2.15)

Where \( \alpha \) and \( R_m \) are the specific resistance of cake and that of the filter medium respectively. After integration, equation (2.15) turns out a straight line, when \( t/v \) is plotted against \( v \) with slope \( = \frac{\mu ac}{2PA^2} \) and the intercept \( = \frac{R_m \mu}{PA} \). There are two major problems in using Coackley’s apparatus, since it is based on Carman’s theory, it requires the plot of \( t/v \) against \( v \) and this is expected to give a straight line graph. Most often, the points are scattered and the use of ordinary linear regression will be impossible due to the fact that the variables involved are not independent. Coackley, later proposed curvilinear regression analysis but also pointed out that this method requires special statistical expertise and it takes time. On this note, the straight line is drawn by inspection and therefore results to much error to the slope of \( t/v \) against \( v \) which is the bases for evaluating \( r \), the specific resistance. Another problem is that of the effective area of filtration, in earlier work, Coackley (1958) assumed that the effective filtration area of the Buchner funnel was equal to the area of the filter medium, the filter paper. Swanwick, et al (1961), reported that this overestimated the true filtration area by more than 25%. Since the calculation of the specific resistance to filtration is dependent on the square of the filtration area in Carman’s equation.
In 1974 Anazodo, refused to give support to Carman’s equation due to its formulation point of view. He argued that the approximation of compressible filter cakes to rigid bundles of capillary tubes or to non-compressible sand-bed did not make sense. He came up with another basic theoretical equation for the filtration of compressible sludges using FMTL_x L_y L_z dimensional analysis. His approach did not involve poiseuille’s and D’Arcy’s laws. He suggested that the factors that could influence the volume of filtrate effectively are pressure p, Area A, viscosity \( \mu \), solid content C, specific resistance r, and time t. As a matter of fact those terms are found in Carman’s equation.

Anazodo stated thus;

\[
V = P^a A^b \mu^c C^d r^e t^f \quad ................. \quad (2.16).
\]

Where a, b, c, d, e, and f are numerical exponents. All the variables that have significant effect on sludge filtration are therefore reduced to the fundamental quantity of force F, mass M, time T, length L. Length was subdivided into three x, y, z, and represent the three mutually perpendicular axes in space.

Anazodo finally gave a partial equation of the form,

\[
V = \left[ \frac{A^2}{Cr^2} \right]^{1/2} \left[ \frac{A^{1/2} p C^{1/2} r^{1/2} t}{\mu} \right]^f \quad ................. \quad (2.17).
\]

Consequently, there is a universal acceptance of the experimental and theoretical evidence that the time- volume relationship of all constant-pressure filtration show perfect parabolic behaviour. On that bases, Anazodo substituted \( f = \frac{1}{2} \) in equation (2.16) to obtain dimensional equation for sludge filtration as,

\[
V^2 = \frac{P A^{5/2} t}{\mu C^{1/2} r^{1/2}} \quad ................. \quad (2.18).
\]
After the publication of the dimensional equation, White and Gale did not approve it, they argued that;

1) Carman’s equation which predicts that the volume of filtration is directly proportional to the area of filtration is preferable.
2) Anazodo did not justify the prediction that volume of filtrate obtained after a fixed time is proportional to the filtration area to the power of 5/4,
3) Mass and Force can both be assumed as fundamentals only in the unusual circumstance when the physical proportionality between the two is not involved in the terms.
4) Anazodo need not assume \( f = 1/2 \)

White and Gale, suggested that Anazodo’s partial equation should be written thus;

\[
V^2 = P\mu^{-1}A^{2b}t(Gr)^{2b-3} \cdots \cdots \cdots \cdots \cdots \ (2.19)
\]

So that if \( b = 1 \) Carman’s equation is arrived at but if \( b = 5/4 \), Anazodo’s equation is implied.

Anazodo (1974), White and Gale (1975), later agreed that the correct determination of the exponent \( b \), based on theoretical or experimental consideration would guide the choice of filtration equation.

In 1982, Ademiluyi, and others, worked relentlessly to investigate and determine experimentally the value of the exponent \( b \), which related the volume of filtrate to the area of filtration. They used the Buncher funnel technique as described by Coakley and found that the average value of the exponent \( b \), is \( 0.91 \pm 0.02 \), if calculations were made with the total area of filtration and also that the value of \( b \), was equal to \( 1.38 \pm 0.02 \), if the effective area of the filtration
was used in experimental data analysis. Based on their findings, they suggested that the total area of filtration should be used in Carman’s equation and the effective area of filtration should be used in the dimensional equation suggested by Anazodo for sludge filtration at constant pressure.

After the substitution of the exponent b, in the partial dimensional equation Ademiluyi, et al. (1982), suggested the following equations,

\[ V^2 = \frac{P A_t^{1.82} t}{\mu (Cr)^{1.18}} \] ........................ (2.20)

And

\[ V^2 = \frac{P A_{eff}^{2.76} t}{\mu (Cr)^{0.24}} \] ........................ (2.21).

Where \( A_t \) and \( A_{eff} \) are the total area and the effective area of the Buchner funnel respectively. \( V = \) volume of filtrate, \( \mu = \) Dynamic viscosity, \( t = \) time of filtration, \( r = \) specific resistance, \( C = \) cake concentration, \( P = \) pressure.

They concluded that the calculation of the specific resistance to compressible sludges could be done in three possible ways;

1. That Carman’s equation should be used with the filtration area measured as the total area of the Buchner funnel.
2. That the dimensional equation suggested by Anazodo should be used with the filtration area measured as the effective area of the Buchner funnel.
3. That the partial dimensional equation suggested by White and Gale should be used by determining the value of the exponent b, and then obtain a modified Carman’s equation or modified dimensional equation depending on whether the total area or the effective area of the Buchner funnel was used.
Agunwamba and others, (1989), suggested new sludge filtration equation which incorporates the input variable, sludge concentration \( C_0 \); the state variables, specific resistance \( R \); and the output variables, cake concentration \( C \) and filtrate concentration \( C_f \) which originated from material balance and was analysed using regression analysis. An equation of this nature would be useful in the process of optimization since it contains the most important filtration parameters. The material balance equation for sludge filtration process can be stated thus;

\[
\text{Inflow} \quad \text{Outflow} = \text{Rate of Accumulations.}
\]

\[
\text{I} \quad \text{e} \quad Q_0 C_0 - Q C_f = V_f \frac{dC}{dt} \quad \text{............... (2.22)}.
\]

Where \( Q_0 \) is the rate of sludge inflow \( (m^3/s) \); \( Q \) is the rate of volumetric increase of the filtrate \( (m^3/s) \); \( C_0 \) and \( C_f \) are the concentration of the sludge \( (Kg/m^3) \) and the filtrate \( (Kg/m^3) \), respectively. \( dC \) is a small change in the cake concentration \( (Kg/m^3) \).

Assuming a constant rate of filtration,

\[
V_f = Q t \quad \text{......................... (2.23)}.
\]

Where \( t \) is the time taken to obtain the filtrate in seconds.

Ignoring losses,

Volume of sludge filtered = volume of cake + volume of filtration,

\[
V_0 = V_f + V_c \quad \text{................................. (2.24)}.
\]
Also,

\[ V_0 = Q_0 t \] ........................................... \( (2.25) \).

Integrating equation \( (2.22) \) for cases where \( Q_0 \) and \( Q \) are time independent,

\[ CV_f = Q_0 C_0 t - Q C_f t \] ....................... \( (2.26) \).

From equation \( (2.23) \), equation \( (2.25) \) and \( (2.26) \) and eliminating \( Q_0 \) and \( Q \),

\[ C_0(V_f + V_c) = C_f C_f + C V_f \] ................... \( (2.27) \)

Agunwamba,\( (2001) \) suggested that the regression equation connecting the variables \( C \) and \( C_f \) is given as;

\[ C = K_1 C_f + K_2 \] .................................. \( (2.28) \)

Where \( K_1 \) and \( K_2 \) are constants.

Substituting equation \( 2.27 \) and \( 2.28 \) in \( 2.26 \) above,

\[ C_f = \frac{c_0}{1+K_1} + \frac{c_0 c_0}{(1+K_1) V_f} - \frac{K_2}{1+K_1} \] ................... \( (2.29) \)

The constant, \( K_2/(1 + K_1) \) does not affect the optimization of sludge filtration process, hence equation \( 2.29 \) reduces to

\[ C_f^1 = \frac{c_0}{1+K_1} + \frac{c_0 V_0}{(1+K_1) V_f} \] ................... \( (2.30) \)

Where,

\[ C_f^1 = C_f + K_2/( 1 + K_1) \] ................... \( (2.31) \)

Since the specific resistance is an important parameter in sludge filtration, it is introduced in equation \( 2.30 \).
From equation 2.8 following Coackley’s modification,

\[ V_f^2 = \frac{2 PA^2t}{RC\mu} \quad \ldots \quad (2.32) \]

Neglecting the septum resistance, \( R_m \), and \( t \) is the laboratory time measured in seconds. The yield from a filter plant is given as,

\[ Y = \frac{V_fC}{At_c} \quad \ldots \quad (2.33) \]

Where \( t_c \) is the industrial filtration time expressed in hours.

Therefore, equation 2.32 can be expressed further as,

\[ V_f \left( \frac{V_fC}{At_c} \right) = \frac{2 PA \times 3600}{R \mu} \quad \ldots \quad (2.34) \]

From equations 2.33 and 2.34

\[ V_f = \frac{7200 PA}{R \mu Y} \quad \ldots \quad (2.35) \]

The yield \( Y \), is found to be proportional to the concentration of the sludge i.e

\[ Y = C_0K_0 \quad \ldots \quad (2.36) \]

Substituting for \( Y \) Equation 2.35 becomes,

\[ V_f = \frac{7200 PA}{R \mu C_0K_0} \quad \ldots \quad (2.37) \]

Hence equation 2.30 can be expressed as

\[ C_f^1 = \frac{C_0}{1+K_1} + \frac{C_0V_0R\mu C_0K_0}{7200(1+K_1)PA} \quad \ldots \quad (2.38) \]

But \( V_c = A \, d \), and \( d \) is thickness of the cake.
Equation 2.38 is simplified to

\[ C_f^1 = \frac{c_0}{1+K_1} + \frac{c_0^2 K_0 d R}{7200 (1+K_1)} \] ............ (2.39)

Equation 2.39 is the desired equation. It connects the output variable \( C_f \), the state variable \( R \) with the input variables \( c_0 \). It can be rewritten as

\[ \frac{C_f^1}{c_0} = K_3 + K_4 (c_0 K_0 R) \] ............ (2.40)

Where,

\[ K_3 = \frac{1}{(K_1 + 1)} \] ............ (2.41)

\[ k_4 = \frac{\mu d}{7200 \, p (1+K_1)} \] ............ (2.42)

\( K_3 \) and \( K_4 \) are regarded as constant for constant pressure filtration. If however, this is not the case, for constant sludge concentration, equation 2.39 becomes,

\[ C_f^1 = K_5 + K_6 \left( \frac{K_0 \, R}{p} \right) \] ............ (2.43)

Where,

\[ K_5 = \frac{c_0}{(1+K_1)} \] ............ (2.44)

\[ K_6 = \frac{c_0^2 \, \mu \, d}{7200 (1+K_1)} \] ............ (2.45)

The specific resistance is obtained from the relationship,

\[ \frac{c_0 V_0}{V_f} = \frac{c_0^2 K_0 d R \, \mu}{c_0 K_0 d V_f \, \mu} \] ............ (2.46).
The plot of \( \frac{C_f^1}{C_0} \) versus \( RC_0 K_0 \) in this equation, \( \frac{C_f^1}{C_0} = \frac{1}{65.9} + 1.16 \times 10^{-15} \), \( RC_0 K_0 \),

Will give a straight line while a plot of \( C_f^1 \) versus \( \frac{R K_0}{P} \) in this equation;

\( C_f^1 = 5.688 + 4.098 \times 10^{-10} \frac{R K_0}{P} \) will also yield a straight line.

Many factors which significantly affect the filtration process are quantitatively incorporated into the basic filtration equation, such as pressure, concentration, area of filtration, viscosity, but there are also some factors that are unquantifiable but play significant role in filtration process as they affect the specific resistance parameter.

Dick, (1974) showed that the shape of sludge particles affect specific resistance. Also contact time between the sludge and coagulant and the sequence of addition of coagulant also influence the specific resistance parameter. Other unquantifiable physical factors affecting the specific resistance parameter include ageing, shear filter medium, hydrolysis and rainfall. Some of these factors are apparently difficult to control in sludge dewatering investigations and they do not readily come to mind and yet their influences on the specific resistance may be significant. In view of the above and also the various units of measurement associated with \( r \), the specific resistance (the measure of sludge filterability) and the attendant loss of benchmarks (Ademiluyi, et al, 1983, Christensen, 1983), a new parameter referred to as sludge dewaterability number (SDN) was proposed by Ademiluyi and others in 1987. He discovered that including the factors enumerated above, that the sludge filtration parameter is dependent not only on the equipment design but also on sludge pre-treatment before dewatering. He therefore stated that;
\[ \text{SDN} = \frac{\Delta H (C_0 - C_f)}{V_i C_c t} + \frac{H_0}{V_it} \quad \text{............... (2.47).} \]

Where \( C_0 \) = initial concentration of sludge; \((Kg/m^3)\).

\( C_f \) = filtrate concentration, \((Kg/m^3)\).

\( V_i \) = approach velocity, \((m/s)\).

\( C_c \) = cake concentration, \((Kg/m^3)\).

\( H_0 \) = initial headloss \((m)\).

\( \Delta H \) = change in headloss, \((m)\).

The \( C_f \) and \( C_c \) in the equation above show that the SDN considers essentially not only the ease by which filtrate is collected but also the ease by which the cake is formed as well as the quality of the cake. It equally takes into account flow rate headloss, filtrate concentration, cake concentration, density, sludge concentration and the time of filter run.

The flow rate is an important factor to account for because a sludge that is filterable is expected to pass through the cake easily and at a short time. The cake concentration is equally important since dewatering process is aimed at not only removing the suspensions but also to concentrate the slurry into cake. Low values of SDN usually imply high sludge filterability and a higher cake quality.

In 1991 Ademiluyi, suggested the use of capillary suction time (CST) apparatus in evaluating SDN. He opined that the equation can be used to investigate the effect of conditioner on sludge filterability.

Rearranging and modifying the SDN equation above, he showed that,
\[ t_1 = \frac{1}{SDNV_1} \left[ \frac{\Delta H_1 C_0}{C_{c_1}} + H_0 \right] \] ........................................ (2.48).

If \( C_f \cong 0 \), and assuming there is no loss of solid particle through filtration then,

\[ t_2 = \frac{1}{SDNV_2} \left[ \frac{\Delta H_2 C_0}{C_{c_2}} + H_0 \right] \]

\[ CST_{sludge} = \frac{1}{SDN} \left[ \frac{\Delta H_2 C_0}{C_{c_2} V_2} + \frac{H_0}{V_2} \right] \left[ \frac{\Delta H_1 C_0}{C_{c_1} V_1} + \frac{H_0}{V_1} \right] \] ........... (2.49).

Considering water to be pure \( C_0 = 0, \ C_c = 0, \ C_f = 0 \)

\[ CST_{water} = t_2 - t_1 = \frac{H_0}{SDN} \left[ \frac{1}{V_2} - \frac{1}{V_1} \right] \]

\( CST \), is the value difference between the \( CST \) of sludge and that of pure water.

This then implies that, \( CST = CST_{sludge} - CST_{water} \)

\[ = \frac{c_0}{SDN} \left[ \frac{\Delta H_2}{V_2 c_{c_2}} - \frac{\Delta H_1}{V_1 c_{c_1}} \right] \] ........... (2.50)

Taking \( log \) of both sides we have,

\[ \log CST = \log \frac{c_0}{SDN} \log \left[ \frac{\Delta H_2}{V_2 c_{c_2}} - \frac{\Delta H_1}{V_1 c_{c_1}} \right] \] ...................... (2.51)

Dry weight of solid in the sludge \( (W_s) = \) Dry weight of solid in the cake,

But \( C_0 = \frac{W_s}{V_s} \), and \( C_c = \frac{W_s}{V_c} \)

Where \( V_s = \) volume of sludge and \( V_c = \) volume of cake,

\[ \therefore \frac{c_0}{c_c} = \frac{V_c}{V_s} \]
Also \( V_c = V_s - V_f \)

Therefore \( C_c = \frac{W_s}{V_s - V_f} \) ............................ (2.52)

Substituting equation 2.52 into equation 2.51

We have \( \log CST = \frac{W_s}{SDN V_s} + \emptyset \)

And \( \emptyset = \log(V_s - V_f) \left( \frac{\Delta H_2}{V_2 W_{s2}} - \frac{\Delta H_1}{V_1 W_{s1}} \right) \)

Considering different sludges, a plot of \( \log \frac{W_s}{SDN V_s} \) against \( \log CST \) should give a slope of 1 (unity) and a straight line.

**2.23 THEORY BASED ON COMPRESSION OF SLUDGE (PERMEABILITY DATA)**

Owing, to the poor correlation found between theoretical and practical results on rotary vacuum filter using the equation suggested by Ruth, Halff in 1952, developed an approach based upon Terzaghi’s, theoretical treatment of the consolidation of soils. He suggested and proposed the hypothesis that the driving force varies with time and distance. The approach is different from filtration equation previously developed by Carman and Ruth.

The derivation of Halff’s equation, establishes the fact that filtration follows the parabolic relationship until the cake reaches 80% filtration or consolidation. He proposed that;

\[ C_f = \frac{\sigma_y L}{\mu S^3} \] ............................ (2.53)
2.24 COMPRESSIBILITY OF SLUDGE.

Compressibility is a significant parameter in filtration process and is defined as decrease in unit volume per unit increase in pressure. It is a measure of the ease with which the solid particles that settles on the filter medium are deformed. When pressure is applied to the filter cake there is every tendency that the solid particle will be densely packed and this results in compression of the cake. If the pressure is sufficient enough, the cake will offer high resistance to the passage of filtrate due to its compressible nature. The rate of water escape depends on the permeability and compressibility of the sludge.

The compressibility coefficient of a sludge is a measure of its compressibility. This parameter was defined by Carman (1934) using the relationship,

\[ r = K P^S \] .................. (2.54)

Ruth equally suggested that, \[ r = r (1 + r_2 P S_c) \] .................(2.55)

The equations are empirical in nature. Carman agreed that his formula has error and showed curves for very compressible material in which deviation from the formula occur. Ruth (1935) showed that his equation does not give room for low compressibility values to be evaluated with any degree of accuracy for rigid cake, \( s = 0 \) and that \( s \) should equal unity.

The perpetual controversy in sludge filtration equation is due to non involvement of Darcy’s law (applicable to rigid cake) to compressible or non rigid materials. In formulation of sludge filtration equation the compressibility attribute must be accounted for.

Carman, Ruth nor Wakeman, gave the qualitative definition of the parameter, compressibility coefficient \( s \). ‘s’ is evaluated by determining the specific
resistance \( r \), at varying pressures. The slope of \( \log r \) against \( \log p \) gives the compressibility coefficient's'. Ademiluyi, et al (1983), reported that this definition does not hold for all degrees of dilutions. He said that for about 50% dilution the theory does not agree with the practice and that it only holds for highly diluted sludges when it is in comparable fluidity state with ordinary water.

Owing, to the significance of this parameter Nebiker, (1968), suggested that factors that affect specific resistance should be investigated to know if they affect compressibility coefficient also. On that note Ademiluyi, Anazodo, and Egbunuwe, (1983), worked tirelessly to investigate the effect of dilution and chemical conditioning on compressibility coefficient using Carman’s equation. They found out and concluded that the decrease in compressibility with increasing dilution as noted within the range of some concentration indicates that within certain systems and concentrations, the empirical formula proposed by Carman \( (r = KP^S) \) may not be obeyed.

A universal definition based on Terzaghi’s (1966), definition of compressibility coefficient cannot be over-emphasized. It is worthy to note that the incorporation of such a parameter into any sludge filtration equation will make that equation unique in its derivation.
CHAPTER THREE

3.0 METHODOLOGY.

3.1 MATERIAL AND EXPERIMENTAL SET UP.

So many techniques exist for dewatering sludges and theses include mechanical as well as non mechanical methods. Mechanical methods include vacuum filters, belt filter press, centrifuge, etc. While non mechanical methods include the drying bed, lagoons, oxidation ditches and ponds, etc. This research work makes use of sand drying bed option for sludge filtration.

![Figure 3.1: Schematic Diagram of Sand Drying](image)

The pilot plant (the drying bed) as shown in figure 3.1 is made of 12mm thick steel plate, welded together to form the walls of the drying bed. The length is
1.2m long, with width of 0.75m and a depth of 0.80m. The total filtration area is 0.9m². The base of the drying bed was perforated and filtrate was collected through a drain pipe into the measuring cylinder. A measuring tape of plastic material was attached to the wall of the bed on one side to read off the change in sludge depth. On the perforated bottom, coarse gravel of 0.2m thick was added and right on top of it 0.2m thick of clean sand was levelled throughout the bed to form a finished surface. The experiment was set up at the wastewater treatment plant at the University Of Nigeria, Nsukka.

3.2 EXPERIMENTAL METHODOLOGY.

Sewage sludge was collected from the Imhoff tank, a primary sedimentation tank at the University of Nigeria, Nsukka with a bucket. The wastewater from this tank are the product of all the waste from different hostels, different laboratories such as chemistry, pharmacy and microbiology laboratories and the staff quarters as well.

The sludge was collected at a depth below the surface and stirred intensively with a stick to break the floc. It was then poured into the drying bed to a height of 0.25m. As this was done filtrate from the sludge started percolating through the sand bed by gravitational movement of water downwards. Equally, water was lost by evaporation from the surface of the sludge.

Four thermometers were used to measure the sludge temperature and the atmospheric temperature as well. They were inserted at the four corners of the drying bed to capture any difference in temperature of the sludge. Measurements were taken at an interval of two hours on the first day and 24 hours interval on the subsequent days.

Readings were taken by reading off after a time interval,
1. The volume of filtrate in the measuring cylinder after a time interval of two hours on the first day and 24 hours on the subsequent days.

2. The time taken for the filtrate to accumulate within the time limits stated above with the aid of a stop watch.

3. Temperature of the sludge as well as the atmospheric temperature with the thermometers.

4. The change in the sludge height using the measuring tape on the wall of the drying bed.

### 3.3 Filtration Pressure (P).

Since we are dealing with natural filtration process, the use of drying bed, the pressure under consideration is hydrostatic pressure as this is what is exerted by liquid when it is at rest. Consequently, the pressure changes as it is being influenced by the density of the liquid and gravity of the area. The filtration pressure is therefore,

\[ p = \rho gh \]

Where,

- \( p \) = hydrostatic pressure (N/m²)
- \( \rho \) = density of water (Kg/m³)
- \( h \) = height of liquid (m)
- \( g \) = acceleration due to gravity (m²)

### 3.4 Dynamic Viscosity (\( \mu \))

The viscosity of the filtrate was taken to be that of water since the filtrate was clear. The dynamic viscosity of water was calculated using,
\[ \mu = 0.0168 \times \rho \times T^{-0.88} \]

Where, \( \mu \) = dynamic viscosity (N.s/m\^2)
\( T \) = Temperature of water (°C)
\( \rho \) = density of water (Kg/m\^3)

3.5 Area of Filtration (A).

The total area covered by the sand bed was considered as the total area and can be calculated as,

\[ \text{Area} = \text{Width} \times \text{Length} \]

3.6 Moisture Content.

Clean dry moisture cans were used, the cans were labelled and weighed without sample and the values recorded. Then a certain volume of sludge sample was poured in the cans and was also weighed and the value noted. The can containing the wet sludge is then placed in an oven at a temperature of 105 °C for 24 hours with the lid of the can removed. After oven dry the lid was replaced by covering the can that is containing the dry sludge and it was reweighed, the new weight was recorded. There was a loss in weight in all the cans.

The moisture content is calculated thus,

\[ \% \text{ Moisture Content} = \left( \frac{\text{Weight of wet sludge} - \text{Weight of dry sludge}}{\text{Weight of dry sludge}} \right) \times 100 \]

3.7 Effect of Chemical Conditioning on Sewage sludge.

To investigate the effect of conditioner on sludge filterability the traditional ferric chloride (\( \text{FeCl}_3 \)) was used at different concentration to a given volume of
sludge. Different concentration of 10g, 20g, 30g, 40g, and 50g were respectively added and mixed properly with 10liters of sludge. Each filtration circle took 25minuite after which the volume of filtrate was measured and recorded. The height of the sludge in the drying bed was also noted as well as the temperature for each filtration run.

The effect of dilution was investigated by adding different dosage of distilled water as follows 1litre, 2litres, 3litres, 4litres, and 5litres to 10litres of sludge each forming a different filtration circle .To produce a filterable sludge at different solid content, the 10litres sludge for each filtration run was conditioned with 20g of ferric chloride. A certain volume of the conditioned sludge was oven dried at 105°C following the laid down procedure to enable the determination of the solid content.

3.8 DERIVATION OF FILTRATION EQUATION USING DIMENSIONAL ANALYSIS

Dimensional analysis as defined by Rajput,(1998) is a mathematical technique which makes use of the study of the dimensions for solving several engineering problems. Each physical phenomenon can be expressed by an equation giving relationship between quantities which may be dimensional and non-dimensional. It helps in determining a systematic arrangement of the variables in the physical relationship, combining dimensional variables to form non – dimensional parameters.

Dimensional analysis has become an important tool for analysing fluid flow problems. It is especially useful in presenting experimental results in a concise form. It is also used to test the dimensional homogeneity of any equation of fluid
motion. It has the advantage of expressing the functional relationship between the variables in dimensionless terms.

Dimensional analysis has its limitations in that it does not give any clue regarding the selection of variables. On that note, if any variable is wrongly taken the resulting functional relationship is erroneous. The complete information is not provided by dimensional analysis it only indicates that there is some relationship between the parameters.

Since no chemical reaction is assumed to take place during sludge filtration, the process can be assumed to constitute a physical system and dimensional analysis can therefore be used in deriving an equation to describe the system.

During any filtration cycle, and when cake has been formed such that the septum resistance is negligible, the following parameters affect the volume of filtrate, Net filtration pressure, $P$; Time taken to obtain the filtrate, $t$; Area of filtration, $A$; Viscosity of filtrate, $\mu$; Mass of dry cake deposited per unit volume of filtrate, $C$; Density of filtrate, $\rho_f$; Density of filter cake, $\rho_f$; Shape, orientation and distribution of the solid particles in the cake, $K$; Specific surface of the cake, $s$; and the porosity of cake, $e$;

However, in deriving the equation only the effective terms are incorporated. The summary of the variables are as shown below.
**Table 3.0** summary of dimensional formula of the effective variables affecting filtration.

<table>
<thead>
<tr>
<th>Physical Variables</th>
<th>Symbols</th>
<th>Fundamental system (LMT)</th>
<th>Fundamental system (FMTL&lt;sub&gt;x&lt;/sub&gt;L&lt;sub&gt;y&lt;/sub&gt;L&lt;sub&gt;z&lt;/sub&gt;)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Volume of filtrate</td>
<td>V</td>
<td>L&lt;sup&gt;3&lt;/sup&gt;</td>
<td>L&lt;sub&gt;x&lt;/sub&gt;L&lt;sub&gt;y&lt;/sub&gt;L&lt;sub&gt;z&lt;/sub&gt;</td>
</tr>
<tr>
<td>Filtration Time</td>
<td>t</td>
<td>T</td>
<td>T</td>
</tr>
<tr>
<td>Filtration Area</td>
<td>A</td>
<td>L&lt;sup&gt;2&lt;/sup&gt;</td>
<td>L&lt;sub&gt;x&lt;/sub&gt;L&lt;sub&gt;y&lt;/sub&gt;L&lt;sub&gt;z&lt;/sub&gt;</td>
</tr>
<tr>
<td>Mass of dry cake per unit volume</td>
<td>C</td>
<td>ML&lt;sup&gt;-3&lt;/sup&gt;</td>
<td>ML&lt;sub&gt;x&lt;/sub&gt;&lt;sup&gt;-1&lt;/sup&gt;L&lt;sub&gt;y&lt;/sub&gt;&lt;sup&gt;-1&lt;/sup&gt;L&lt;sub&gt;z&lt;/sub&gt;&lt;sup&gt;-1&lt;/sup&gt;</td>
</tr>
<tr>
<td>Net filtration pressure</td>
<td>P</td>
<td>ML&lt;sup&gt;-1&lt;/sup&gt;T&lt;sup&gt;-2&lt;/sup&gt;</td>
<td>FL&lt;sub&gt;x&lt;/sub&gt;&lt;sup&gt;-1&lt;/sup&gt;L&lt;sub&gt;y&lt;/sub&gt;&lt;sup&gt;-1&lt;/sup&gt;</td>
</tr>
<tr>
<td>Viscosity of filtrate</td>
<td>μ</td>
<td>ML&lt;sub&gt;z&lt;/sub&gt;&lt;sup&gt;-1&lt;/sup&gt;T&lt;sup&gt;-1&lt;/sup&gt;</td>
<td>FTL&lt;sub&gt;z&lt;/sub&gt;&lt;sup&gt;2&lt;/sup&gt;</td>
</tr>
<tr>
<td>Specific resistance of filter cake</td>
<td>R</td>
<td>LM&lt;sup&gt;-1&lt;/sup&gt;</td>
<td>L&lt;sub&gt;z&lt;/sub&gt;M&lt;sup&gt;-1&lt;/sup&gt;</td>
</tr>
</tbody>
</table>

Accordingly, Anazodo (1974) differentiated length into three x, y, z and they represent the three mutual perpendicular axes in space. He equally suggested that r, the specific resistance may be used to replace all the cake properties and the dimensional equation can be written thus,

Mathematically,

\[ V = P^a A^b C^c \mu^d R^e t^f ..... \]  \hspace{1cm} (3.10)

In terms of \( FML_x L_y L_z T \)

\[ \text{Pressure, } p = \text{Force/Area, } \equiv \frac{F}{L_x L_y} \]
Area of drying bed, $A = \text{Length} \times \text{Width} \equiv L_x L_y$

Concentration of cake, $C = \text{Mass/Volume} = M/L_x L_y L_z$

Viscosity, $\mu = \text{Force} \times \text{Time/Area} \equiv FT/L_z L_z$

Time of filtration, $T = T \equiv T$

Specific Resistance of cake, $R = \text{Length/Mass} \equiv L_z/M$

Applying dimensions,

$$L_x L_y L_z = (FL_x^{-1} L_y^{-1})^a (L_x L_y)^b (ML_x^{-1} L_y^{-1} L_z^{-1})^c (FTL_z^{-2})^d (M^{-1} L_z)^e (T)^f.$$  

For $F$: $0 = a + d$ \hspace{1cm} (3.11)

For $L_x L_y$: $1 = -a + b - c$ \hspace{1cm} (3.12)

For $L_z$: $1 = -c - 2d + e$ \hspace{1cm} (3.13)

For $M$: $0 = c - e$ \hspace{1cm} (3.14)

For $T$: $0 = d + f$ \hspace{1cm} (3.15)

From the above expression of equations we have five equations with six unknown variables.

From equation (3.10) $a = -d$

And from equation (3.14) $c = e$

Substituting in equation (3.13) we have,

$$1 = -c - 2d + e$$

Since $c = e$ and $d = -a$
\[ \therefore 1 = -e - 2(-a) + e \]

\[ 1 = -e + 2a + e \]

\[ 1 = 2a \]

\[ a = \frac{1}{2} \]

Substituting \( a = \frac{1}{2} \) in equation (3.11) we have,

\[ 0 = a + d \]

\[ 0 = \frac{1}{2} + d \]

\[ \therefore d = -\frac{1}{2} \]

Substituting \( d = -\frac{1}{2} \) in equation (3.15) we obtain,

\[ 0 = d + f \]

\[ 0 = -\frac{1}{2} + f \]

\[ \therefore f = \frac{1}{2} \]

Since \( c = e \) then, applying it in equation (3.12) above

\[ 1 = -a + b - c \]

\[ 1 = -1/2 + b - e \]

\[ 1 + 1/2 = b - e \]

\[ 3/2 = b - e \]

\[ \therefore b = 3/2 + e \]
Therefore, $a = 1/2, b = 3/2 + e , c = e , d = -1/2, f = 1/2$

substituting in the partial equation,

$$V = P \frac{1}{2} A^{(\frac{3}{2}+e)} e^{\mu - \frac{1}{2}} R e t^\frac{1}{2}$$

Solving we obtain;

$$V = \left( \frac{P t}{\mu} \right)^{\frac{1}{2}} A^{\frac{3}{2}} e^{\mu} R e t^\frac{1}{2} \tag{3.16}$$

$$V = \left( \frac{P t}{\mu} \right)^{\frac{1}{2}} A^{\frac{3}{2}} (ACR)^e \tag{3.17}$$

$$V = \left( \frac{P t A^3}{\mu} \right)^{\frac{1}{2}} (ACR)^e \tag{3.18}$$

$$V = t^\frac{1}{2} \left( \frac{P A^3}{\mu} \right)^{\frac{1}{2}} (ACR)^e \tag{3.19}$$

$$\frac{V}{t^\frac{1}{2}} = \left( \frac{P A^3}{\mu} \right)^{\frac{1}{2}} (ACR)^e \tag{3.20}$$

$$\frac{t^\frac{1}{2}}{V} = \left( \frac{P A^3}{\mu} \right)^{-\frac{1}{2}} (ACR)^{-e} \tag{3.21}$$

Squaring both sides of the equation we obtain,

$$\frac{t}{V^2} = \left( \frac{P A^3}{\mu} \right)^{-1} (ACR)^{-2e} \tag{3.22}$$

$$\frac{t}{V} = V \left( \frac{\mu}{PA^3} \right) (ACR)^{-2e} \tag{3.23}$$
If \( \frac{t}{v} \) is plotted against solid content, \( C \), it gives a linear relationship and hence shows that the volume \( V \) of filtrate is proportional to the solid content \( C \).

Figure 3.2, showing graph of \( t/V \) against solid content.

Equating the powers we have,

\[-2e = 1\]

\[ e = \frac{1}{2} \] .....................................................(3.24)

Substituting, \( e = -\frac{1}{2} \) in equation (3.23)

\[ \frac{t}{V} = V \left( \frac{\mu}{PA^3} \right) (ACR)^{-2 \times -1/2} \] ..........................(3.25)

\[ \therefore \frac{t}{V} = V \left( \frac{\mu CR}{PA^2} \right) \] ..........................(3.26)

But \( t/v \) is a function of \( \frac{V\mu CR}{PA^2} \)
ie \( \frac{t}{V} = f\left(\frac{\mu CR}{pA^2}\right) \)

By observing the experimental plot of \( t/V \) versus \( V \), equation (3.26) should be re-written as,

\[
\frac{t}{V} = V \left(\frac{\mu CR}{\rho g h A^2}\right) + \beta 
\]

(3.26b)

Where \( \beta \) = intercept of the plot of \( t/V \) against \( V \).

Since we are working on Natural drying bed, the pressure (\( P \)) under consideration is hydrostatic,

That is, \( P = \rho gh \).

Applying \( p = \rho gh \) in equation (3.26b) we obtain,

\[
\frac{t}{V} = V \left(\frac{\mu CR}{\rho g h A^2}\right) + \beta 
\]

(3.27)

Also \( C = \frac{w_d}{V_s} \):

\[
\therefore \frac{t}{V} = V \left(\frac{\mu R w_d}{\rho g h A^2 V_s}\right) + \beta 
\]

(3.27b)

If \( \frac{t}{V} \) is plotted against \( V \), the slope say \( b \) of the straight line is

Slope \( b = \frac{\mu R w_d}{\rho g h A^2 V_s} \) 

(3.28)

Therefore, the specific resistance \( R \), is calculated thus,

\[
R = b \left(\frac{\rho g h A^2 V_s}{\mu w_d}\right) 
\]

(3.29)

Where \( R \) = Specific Resistance (m/kg) \quad A = \text{Filtration Area (m}^2\text{)}

\( \mu \) = Dynamic viscosity (N. s/m\(^2\)) \quad C = \text{Solid content (kg/m}^3\text{)}

\( \rho gh \) = Hydrostatic pressure (N/m\(^2\)) \quad b = \text{Slope (s/m}^6\text{)}

\( w_d \) = Weight of dry cake (Kg) \quad V_s = \text{Volume of sludge (m}^3\text{)}
CHAPTER FOUR

4.10 RESULTS AND DATA ANALYSIS.

Natural filtration process that makes use of sand drying bed was used to reduce the volume of unconditioned sludge by dewatering. This was achieved by gravity drainage through the sludge mass and supporting sand and by evaporation from the surface exposed to the air. As the filtration process began, it was observed that the filtrate was high on the first day and this may be attributed to the low resistance offered by the sand bed alone. As the filtration process proceeds, the thickness of the cake keeps building up and as a result there is a reduction in the volume of filtrate and this is due to the combined effect of the resistance offered by the filter septum and the cake so formed. As the sludge height decreases there is a corresponding increase in the hydrostatic pressure.

Observations were made and data collected every two hours for 12 hours on the first day of the experiment and 24 hours interval for subsequent observations for a duration of 20 days. The data obtained were used in testing and validating the derived dimensional equation. The data and graph analysis are as shown below;
Table 4.10 Filtration of unconditioned sludge on day one of the experiment at 2 hours interval.

<table>
<thead>
<tr>
<th>Volume $V (m^3)$</th>
<th>Time t(s)</th>
<th>Height of sludge $H (m)$</th>
<th>Hydrostatic pressure $\rho g H (N/m^2)$</th>
<th>Dynamic viscosity $\mu (N.s/m^2)$</th>
<th>Density of H$_2$O at different temp. $\rho g (Kg/m^3)$</th>
<th>Solid content $c (Kg/m^3)$</th>
<th>t/V(s/m$^3$)</th>
<th>V$^3$V$^2$/V(s)</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.054729</td>
<td>7200</td>
<td>0.058</td>
<td>566.84</td>
<td>0.8916</td>
<td>996.232</td>
<td>16.02</td>
<td>131557.31</td>
<td>0.002995</td>
</tr>
<tr>
<td>0.062659</td>
<td>14400</td>
<td>0.071</td>
<td>693.58</td>
<td>0.8512</td>
<td>995.796</td>
<td>41.29</td>
<td>229815.35</td>
<td>0.003926</td>
</tr>
<tr>
<td>0.071967</td>
<td>21600</td>
<td>0.079</td>
<td>771.62</td>
<td>0.8386</td>
<td>995.646</td>
<td>77.71</td>
<td>300137.56</td>
<td>0.005179</td>
</tr>
<tr>
<td>0.079592</td>
<td>28800</td>
<td>0.083</td>
<td>810.78</td>
<td>0.8642</td>
<td>996.944</td>
<td>94.86</td>
<td>361845.41</td>
<td>0.006335</td>
</tr>
<tr>
<td>0.085967</td>
<td>36000</td>
<td>0.087</td>
<td>850.13</td>
<td>0.8916</td>
<td>996.232</td>
<td>113.46</td>
<td>418765.34</td>
<td>0.007390</td>
</tr>
<tr>
<td>0.092097</td>
<td>43200</td>
<td>0.100</td>
<td>976.85</td>
<td>0.9522</td>
<td>996.783</td>
<td>118.0</td>
<td>469070.65</td>
<td>0.008482</td>
</tr>
<tr>
<td>0.447011</td>
<td></td>
<td>778.3</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td>1911191.8</td>
<td>0.034306</td>
</tr>
</tbody>
</table>

Initial height = 0.25m, Area = 0.9m$^2$, Hydrostatic pressure = 778.3 N/m$^2$, Dynamic viscosity = 0.8816 N.s/m$^2$, Solid content = 76.89 Kg/m$^3$, $R = 0.81716 \times 10^8$ m/Kg, Slope $b = 8.787 \times 10^6$ s/m$^6$.

The slope $b$ was analysed using regression, i.e. $b = \frac{n \sum \frac{v(t/V)}{\sum V} - \sum V \sum t/V}{n \sum V^2 - (\sum V)^2}$.

The specific resistance $R = \left( \frac{\rho g h A^2}{\mu c} \right) b$. 

The specific resistance R = \left( \frac{\rho g h A^2}{\mu c} \right) b
Figure 4.10 Plot of $t/v$ against volume of filtrate on day one of the experiment.

Table 4.11 Filtration of unconditioned sludge obtained after 24 hours for 5 days.

<table>
<thead>
<tr>
<th>Volume $V$(m$^3$)</th>
<th>Time $t$(s)</th>
<th>Height of sludge $H$(m)</th>
<th>Hydrostatic pressure $\rho g H$($N/m^2$)</th>
<th>Dynamic viscosity $\mu$($N.s/m^2$)</th>
<th>Density of $H_2O$ at different temp. $\rho g/m^3$</th>
<th>Solid content $c(Kg/m^3)$</th>
<th>$t/V$(s/m$^3$)</th>
<th>$V^2$(m$^6$)</th>
<th>$V^4/t/V$(s)</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.12666</td>
<td>86400</td>
<td>0.162</td>
<td>1584.32</td>
<td>0.9688</td>
<td>996.914</td>
<td>139.1</td>
<td>682141.2</td>
<td>0.01604</td>
<td>86400</td>
</tr>
<tr>
<td>0.13186</td>
<td>172800</td>
<td>0.204</td>
<td>1995.83</td>
<td>0.10222</td>
<td>997.296</td>
<td>413.32</td>
<td>1310480.8</td>
<td>0.01739</td>
<td>172800</td>
</tr>
<tr>
<td>0.13361</td>
<td>259200</td>
<td>0.206</td>
<td>2014.62</td>
<td>0.9688</td>
<td>996.914</td>
<td>1095.91</td>
<td>1939974.6</td>
<td>0.01785</td>
<td>259200</td>
</tr>
<tr>
<td>0.13427</td>
<td>345600</td>
<td>0.207</td>
<td>2023.31</td>
<td>0.9060</td>
<td>996.373</td>
<td>812.70</td>
<td>2573918.2</td>
<td>0.01803</td>
<td>345600</td>
</tr>
<tr>
<td>0.13516</td>
<td>432000</td>
<td>0.210</td>
<td>2054.53</td>
<td>1.0222</td>
<td>997.296</td>
<td>738.06</td>
<td>3196211.9</td>
<td>0.01827</td>
<td>432000</td>
</tr>
<tr>
<td>0.66156</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Area = 0.9m$^2$, Hydrostatic pressure = 1934.52 N/m$^2$, Dynamic viscosity = 0.9776 N.s/m$^2$, Slope $b$ = $3.053206 \times 10^8$ s/m$^2$, $R = 7.6488x 10^8$ m/kg, $W_d = 143.96Kg$, $V_s = 0.225m^3$
Table 4.12 Filtration of unconditioned sludge obtained after 24 hours for 5 days.

<table>
<thead>
<tr>
<th>Volume V(m³)</th>
<th>Time t(s)</th>
<th>Height of sludge H(m)</th>
<th>Hydrostatic pressure ( \rho g h (N/m^2) )</th>
<th>Dynamic viscosity ( \mu (N.s/m^2) )</th>
<th>Density of H₂O at different temp. ( Kg/m^3 )</th>
<th>Solid content ( c(Kg/m^3) )</th>
<th>( t/V(s/m^3) )</th>
<th>( V^2(m^6) )</th>
<th>( V^*t/V(s) )</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.13614</td>
<td>518400</td>
<td>0.210</td>
<td>2053.74</td>
<td>0.9688</td>
<td>996.914</td>
<td>179.28</td>
<td>3807844.9</td>
<td>0.01854</td>
<td>518400</td>
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<tr>
<td>0.13705</td>
<td>604800</td>
<td>0.210</td>
<td>2054.53</td>
<td>1.0222</td>
<td>997.296</td>
<td>453.5</td>
<td>4412988.0</td>
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<tr>
<td>0.13745</td>
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<td>0.210</td>
<td>2054.53</td>
<td>1.0222</td>
<td>997.296</td>
<td>1136.09</td>
<td>5028737.7</td>
<td>0.01889</td>
<td>691200</td>
</tr>
<tr>
<td>0.13779</td>
<td>777600</td>
<td>0.211</td>
<td>2064.31</td>
<td>1.0222</td>
<td>997.296</td>
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<td>0.13827</td>
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<td>997.296</td>
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<td></td>
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<td></td>
<td>680</td>
<td>25141585</td>
<td>0.09432</td>
<td>3456000</td>
</tr>
</tbody>
</table>

Area = 0.9m², Hydrostatic pressure = 2058.29 N/m², Dynamic viscosity = 1.01152 N.s/m², Slope b = 3.54293 x 10⁸ s/m², R = 8.5876 x 10⁸ m/kg, \( w_d = 162Kg \), \( V_s = 0.225m^3 \)

Table 4.13 Filtration of unconditioned sludge obtained after 24 hours for 5 days.

<table>
<thead>
<tr>
<th>Volume V(m³)</th>
<th>Time t(s)</th>
<th>Height of sludge H(m)</th>
<th>Hydrostatic pressure ( \rho g h (N/m^2) )</th>
<th>Dynamic viscosity ( \mu (N.s/m^2) )</th>
<th>Density of H₂O at different temp. ( Kg/m^3 )</th>
<th>Solid content ( c(Kg/m^3) )</th>
<th>( t/V(s/m^3) )</th>
<th>( V^2(m^6) )</th>
<th>( V^*t/V(s) )</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.13881</td>
<td>950400</td>
<td>0.212</td>
<td>2073.30</td>
<td>0.9688</td>
<td>996.914</td>
<td>219.28</td>
<td>6846768.9</td>
<td>0.01927</td>
<td>950400</td>
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<tr>
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<td>1036800</td>
<td>0.212</td>
<td>2074.10</td>
<td>1.0222</td>
<td>997.296</td>
<td>493.5</td>
<td>7445066.8</td>
<td>0.01939</td>
<td>1036800</td>
</tr>
<tr>
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<td>0.213</td>
<td>2083.08</td>
<td>0.9688</td>
<td>996.914</td>
<td>1176.09</td>
<td>8044116.6</td>
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<td>0.69751</td>
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<td></td>
<td>2079.32</td>
<td>0.97948</td>
<td></td>
<td>720</td>
<td>40245455.7</td>
<td>0.09731</td>
<td>5616000</td>
</tr>
</tbody>
</table>

Slope b = 3.7030 x 10⁸ s/m², Specific resistance R = 8.84366 x 10⁸ m/kg, Hydrostatic pressure = 2079.32N/m², Dynamic Viscosity = 0.97948N.s/m², Area = 0.9m², Solid Content = 720Kg/m³.
Table 4.14 Filtration of unconditioned sludge obtained after 24 hours for 5 days.

<table>
<thead>
<tr>
<th>Volume ( V(\text{m}^3) )</th>
<th>Time ( t(\text{s}) )</th>
<th>Height of sludge ( H(\text{m}) )</th>
<th>Hydrostatic pressure ( \rho gh(\text{N/m}^2) )</th>
<th>Dynamic viscosity ( \mu(\text{N.s/m}^2) )</th>
<th>Density of ( \text{H}_2\text{O} ) at different temp. ( c(\text{Kg/m}^3) )</th>
<th>Solid content ( c(\text{Kg/m}^3) )</th>
<th>( t/V(\text{s/m}^3) )</th>
<th>( V^2(\text{m}^2) )</th>
<th>( V^2/t/V(\text{s}) )</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.14010</td>
<td>1382400</td>
<td>0.213</td>
<td>2083.87</td>
<td>1.0222</td>
<td>997.296</td>
<td>5563.85</td>
<td>9867237.7</td>
<td>0.01962</td>
<td>1382400</td>
</tr>
<tr>
<td>0.14030</td>
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<td>0.213</td>
<td>2083.82</td>
<td>0.9688</td>
<td>996.914</td>
<td>9041.25</td>
<td>10468995.01</td>
<td>0.01969</td>
<td>1468800</td>
</tr>
<tr>
<td>0.14040</td>
<td>1555200</td>
<td>0.213</td>
<td>2083.82</td>
<td>0.9688</td>
<td>996.914</td>
<td>14466.0</td>
<td>11084818.3</td>
<td>0.01971</td>
<td>1555200</td>
</tr>
<tr>
<td>0.14040</td>
<td>1641600</td>
<td>0.215</td>
<td>2101.50</td>
<td>0.9060</td>
<td>996.373</td>
<td>36165.0</td>
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<td>0.01971</td>
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</tr>
<tr>
<td>0.84186</td>
<td>1728000</td>
<td>0.215</td>
<td>2101.64</td>
<td>0.9688</td>
<td>996.914</td>
<td>49726.87</td>
<td>12307692.3</td>
<td>0.01971</td>
<td>1728000</td>
</tr>
</tbody>
</table>

Slope \( b = 3.7029 \times 10^8 \text{ s/m}^2 \), Specific resistance \( R = 8.84365 \times 10^8 \text{ m/kg} \), Area = 0.9m\(^2\), Hydrostatic pressure = 2091.13N/m\(^2\), Dynamic Viscosity = 0.9669N.s/m\(^2\), Solid Content = 720Kg/m\(^3\).

Figure 4.11 Plot of specific resistance against Hydrostatic pressure.
Figure 4.12 Plot of specific resistance against Solid Content.

From the graphs (fig. 4.11 and 4.12) show that as the hydrostatic pressure and the solid content increases the specific resistance also increases because as more solids settle the pressure increases and the void ratio decreases there by reducing the volume of filtrate that pass through the cake per time.