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<td>Author 2</td>
<td>THOMAS, D. W.</td>
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Effect of Inter-Modulation and Quasi-Periodic Instability in the Diagnosis of Rolling Element Incipient Defect

The absence of significant peaks at the fundamental rotational frequencies, indicative of a particular defect such as an outer race fault, results in a failure of the power spectrum technique to detect and diagnose mechanical defects characterized by sounds of short duration. Based on the results of an incipient defect diagnosis, explanations are offered as to the cause of the missing fundamental in the power spectrum in terms of two different effects which may operate simultaneously. They are:

1. An average and shift effect which causes a slow migration of the fundamental impact frequency from its computed value; and
2. An inter-modulation effect which translates defect related information to frequency locations unrelated to the fundamental impact frequency.

The paper goes on to show that fault detection and diagnosis are possible despite the absence of a significant peak at the fundamental rotational frequency.

1 Introduction

The condition monitoring of machines from an externally acquired control parameter, like vibration, consists of two phases, fault detection and diagnosis. The advantages of this procedure have been extensively discussed in the literature [1] and various analysis techniques proposed [2]. No technique has found universal acceptability since, in general, diagnostic solutions are problem dependent. Hence a mix of complementary techniques is often recommended.

One of the methods used for diagnostic purposes is the power spectral technique. The attractiveness of this technique in the diagnosis of faults in rotating machines is that a significant amplitude change at the rotational frequencies indicates the presence of specific malfunctions. Roberson [3] reported success in bearing defect detection using these rotational frequencies. On the other hand, Murin [4], Broderick [5], and Tatge [6] reported that bearing defect detection is not possible using the rotational frequencies because of the absence of a significant peak at the fundamental rotational frequencies in the power spectrum.

The problem of low level amplitude at the fundamental impact frequency (FIF) or the missing fundamental in the power spectrum, therefore, deserves further study. In a project which involved the diagnosis of incipient rolling contact fatigue failure, based on the analysis of a Seta vibration signature, it was found that fault detection and diagnosis were possible despite the absence of a significant peak at the fundamental rotational frequency. The discriminant features used to detect a defect was a characteristic doublet structure that was present only in the defective power spectrum. These doublet structures occur at frequencies harmonically unrelated to the FIF, but with a recurrence of this structure at every fundamental impact frequency. The periodic recurrence of the doublet structure was found to carry source identification information.

The absence of a significant peak at the FIF was found to be due to two causes which may operate simultaneously:

1. An average and shift effect produced by a quasi-periodic instability or the rapid variation of the impact rate in the waveform. This may cause a slow migration of the fundamental impact frequency from its computed value.
2. An inter-modulation effect which translates defect related information to frequency locations unrelated to the fundamental impact frequency.

The Seta four ball life testing machine from which the vibration signature was acquired is generally used to study lubricant properties which may affect bearing life and to evaluate the life of rolling elements, Scott [7]. The work reported here was part of a project to correlate the tribological and vibrational aspects of condition monitoring, in particular, to detect incipient upper ball damage before it.
could be detected by oil analysis. The Seta's ability to produce the same type of failure as ball bearings quickly and relatively inexpensively is utilized to obtain data on fatigue failure under rolling contact conditions.

This paper first presents a table of the expected rotational frequencies in the defect Seta power spectrum. Next a simplified block diagram of the analysis scheme is described and practical considerations in the estimation of the Seta data under rolling contact conditions.

2 The Seta Data

Seta time history measurements show that in the incipient stage of defect, where the signal has a quasi-periodic structure, the impact rate is unique and can be used to identify the defective component. For the Seta assembly, the rotational frequencies can be determined analytically by first constructing an equivalent ball bearing configuration from the Seta rolling track measurements and then using a ball-bearing rotational frequency formula, Osuaywu [8]. In this way, the fundamental rotational frequencies for the two test speeds, shown in Table 1, can be obtained.

From Table 1, it is evident that the incident defect events occur at low frequencies.

The original vibrational signals were collected using a Nagra IVS and this recorder was used as the input device for the subsequent analysis. The waveforms were low pass filtered at 50kHz with a 96 dB/octave roll off and sampled at 1.6kHz to avoid aliasing. This 500Hz bandwidth then includes all the rotational frequencies of interest for this analysis. If the amplitude at any of these frequencies deviates significantly from the corresponding amplitude in the good case, then the Seta defect associated with that frequency exists.

In all the tests conducted, the defect was found to initiate as a fatigue spall on the upper rolling track (see Plate 1). Since, in general, defects may be present on one or more Seta components simultaneously, the segment length analyzed should contain at least two periods of the lowest rotational frequency of the Seta assembly (see Table 1). Using this criterion, the segment length analyzed was 320μs resulting in a frequency resolution of 3.125Hz.

Although a sampling frequency of 1.6kHz was used, the power spectrum plots of section 4 extend only to 500Hz in order to present the anti-aliasing filter characteristics. In general, defects may be present on one or more Seta components simultaneously, the segment length analyzed should contain at least two periods of the lowest rotational frequency of the Seta assembly (see Table 1). Using this criterion, the segment length analyzed was 320μs resulting in a frequency resolution of 3.125Hz.

Although a sampling frequency of 1.6kHz was used, the power spectrum plots of section 4 extend only to 500Hz in order to present the anti-aliasing filter characteristics from affecting the amplitudes of the spectral estimates.

Table 1

<table>
<thead>
<tr>
<th>Data Name</th>
<th>Rotational Frequencies in Hz</th>
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<tbody>
<tr>
<td>Shaft Speed</td>
<td>f₁</td>
</tr>
<tr>
<td>Seta 6</td>
<td>32</td>
</tr>
<tr>
<td>Seta 8</td>
<td>50</td>
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3 Description of Overall Analysis Scheme

Figure 1 shows a simplified block diagram of an automatic scheme for diagnosing a Seta defect using spectral techniques. Threshold determining steps and the heuristics used in eliminating spurious periodicities due to cyclic interferences in the power cepstrum have been omitted since they are of no direct interest to the objectives of the paper.

The zero mean discrete time series is windowed using a Hanning window and the log power spectrum is then computed. If the spectrum exhibits the characteristic doublet structure, then the Seta is classified as defective; otherwise it is good. For the defective case, the log power spectrum is further processed as extract its periodicity using the power cepstrum technique. Noll [9], Thomas [10]. Finally, comparison of the extracted impact period with the reference periods (the reciprocals of the rotational frequencies of Table 1) reveals the defective component. To increase the cepstral resolution, the doubled-sided log power spectrum is used in comparing the power cepstrum. It should be noted that the power cepstral length is always one half the data length analyzed and that the power cepstrum is used rather than the cepstrum in order to emphasize the cepstral peaks.

Nomenclature

<table>
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<th>Symbol</th>
<th>Description</th>
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<tbody>
<tr>
<td>f₁</td>
<td>shaft frequency, Hz</td>
</tr>
<tr>
<td>f₂</td>
<td>bottom race frequency, Hz</td>
</tr>
<tr>
<td>f₃</td>
<td>cage frequency, Hz</td>
</tr>
<tr>
<td>f₄</td>
<td>upper ball frequency, Hz</td>
</tr>
<tr>
<td>f₅</td>
<td>inner race frequency, Hz</td>
</tr>
<tr>
<td>f₆</td>
<td>outer ball frequency, Hz</td>
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<tr>
<td>f₇</td>
<td>natural frequency, Hz</td>
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4 Results

Characteristics of the Seta Spectra Under Different Operating Conditions

4.1 Seta 6 Spectrum (331Hz, 600kg). Figure 2 shows a typical defective time series and its corresponding log power spectrum at the first defect growth. It can be seen that the spectrum is characterized by double peaks with a frequency separation of 33Hz superimposed on an underlying structure. This frequency separation of 33Hz corresponds to the fundamental cage frequency, \( f_c \). These enhanced double peaks, called doubles, occur at frequencies harmonically related to the fundamental impact frequency, but with a recurrence of this doublet structure every 72Hz. The doublet structure usually starts from a line corresponding to the shaft fundamental frequency.

No peak occurs at the predicted fundamental impact (frequency) of 72Hz and its harmonics. However, peaks occur very close to and to the right of the predicted values. The set of values (72, 75, 144, 152, 216, 225, etc.), indicates the predicted frequency occupancy and the location of the measured peak to the right of the predicted value. It can be seen that the measured values are harmonically related. Therefore, a shift in the frequency occupancy of the FF from (72-75)Hz results in corresponding shifts in the frequency occupancy of its harmonics. The peak at this displaced FF is small relative to the harmonics of the FF are emphasized.

If Fig. 2 is compared with Fig. 3, which shows the time series and corresponding spectrum for Seta 6 in the good state, the suitability of the doublet peaks for detecting inclusions in the defect detect is strikingly demonstrated. With defect, it can be seen that the peak at the first doublet frequency band is 31.5 dB higher than the corresponding peak in the good spectrum.

4.2 Seta 8 Spectrum (50Hz, 600kg). Figure 4 shows a typical defective Seta 8 waveform and the resulting log power spectrum. The peaks at the fundamental impact frequency of 112.5Hz and its harmonics are small. The dominant peaks in the spectrum correspond to the doubles, starting from a base doublet at 50Hz and occurring every fundamental upper half rotational frequency of 112.5Hz. The doublet peaks have a frequency separation of 33Hz and this corresponds to the cage rotational frequency. Ref. Fig. 4 is compared with Fig. 5, which shows the Seta 8 log power spectrum in the good state. It can be seen that a minimum occurs in the good spectrum at the first doublet frequency band of (162.5-175)Hz. This results in an amplitude difference between the good and defect spectra of 42dB, again demonstrating the great sensitivity of the doublet structure to detect Seta defect. Some of the double peaks in the good spectrum show a frequency
separation of 12.8 Hz, but the overall doublet structure is absent.

In general, over all operating conditions, the defective SetA spectra show characteristic doublet structures with a base corresponding to the shaft frequency, \( f_s \), and a recurrence of this structure every \( f_s \). The fundamental impact frequency corresponds to the (rotational frequency of the upper bolt \( f_{ha} \). In particular, the peak at the first doublet frequency \( f_s + f_{ha} \) has been shown to be very sensitive to defect onset and growth. In this good spectra, a minimum usually occurs at this frequency band. In contrast to the high amplitude doublets, the peak at the fundamental impact frequency is very small and does not always occur at the predicted frequency value, but exhibits a small frequency migration. There is no consistent pattern to the amplitude variations at the harmonics of the \( f_s \), although the peaks at these harmonic frequencies are always greater than that at the fundamental impact frequency.

5 Source Identification Using the Power Cepstrum Technique

It has been shown in section 4 that the log power spectrum of the defective signal possesses the following harmonic structures:

(i) doublet peaks with a spectral separation corresponding to the upper rotational frequency,
(ii) doublet structures with a recurrence of this structure every \( f_{ha} \) (upper bolt rotational frequency).

A Fourier transform of the log power spectrum will cause these spectrally related peaks to combine to give a large peak (rhachimics in the cepstrum at a period (quency) corresponding to the reciprocal of their spectral separation. Thus, the cepstrum displays the largest cepstral peak, and by comparing its frequency to the computed impact rate of a defective SetA component, the rotational frequency of the defective doublet can be determined.

5.1 Power Cepstrum of the SetA Defective Data. Figure 6 shows the log power spectrum of the waveform in Fig. 2 and its power cepstrum. The two peaks at \( f_s \) and \( f_{ha} \) at frequencies 12.5 and 14.5 kHz, respectively, delineate the computed upper bolt impact rate of 13.9 Hz. This indicates the presence of an upper bolt defect and shows that the impact rate is varying over the segment analyzed. Peaks that occur at frequencies that are integer multiples of the frequency of another peak on the cepstrum are considered to be rhachimics. The second largest peak, 3f, occurs at the third harmonic of the impact period. Peaks also occur at the second, sixth, and ninth harmonics of the fundamental impact period. The peak at the caged period (9th rhachimic) is small. A rhachimic relationship exists between the third, sixth, and ninth harmonics of the fundamental impact period. It will therefore be assumed that a basic periodicity exists at 41.4 kHz and this would suggest the presence of a bottom race defect.

5.2 Power Cepstrum of the SetA Defective Data. Figure 7 shows the log power spectrum and power cepstrum of the defective SetA waveform of Fig. 4. Here the largest peak occurs at the cage period of 88.6 ms, indicating a cage defect. The second largest peak occurs at the defect period of 8.8 ms corresponding to an upper bolt defect. It can also be seen that a peak occurs at the bottom race period of 28.8 ms and its rhachimic.

Over all operating conditions, the power cepstrum clearly identifies the upper bolt as defective, but it also suggests the presence of two other defects:

1. a bottom race defect
2. a cage defect

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the fundamental impact frequency occurs in a frequency band 69.4 Hz - 71.12 Hz, enclosing the computed impact frequency of 72 Hz, and that the computed impact frequency is the center frequency or mean around which the actual impact frequencies vary. Thus the location of the FIF would migrate within a narrow band of frequencies as found in Section 4.1 and from measurements on the waveforms.

Hence the smallness of the peak at the fundamental impact frequency of the test spectra can be attributed to the quasi-periodic nature of the impacts which cause the impact periods to vary rapidly in the waveform and give rise to double peaks in the power spectrum at a quasiperiod corresponding to the impact period. This has the effect on the Seta spectra of causing a slow migration of the fundamental impact frequency within a narrow frequency band that may or may not be centered on the fundamental impact frequency, and of averaging over this frequency band the power that would otherwise have been concentrated at the impact frequency. This averaging effect reduces the power at the fundamental impact frequency. The shift in the fundamental impact frequency is magnified in the harmonics because of the multiplicative effect involved. The result is to shift the harmonics to higher frequencies thereby destroying their harmonic relationship with the fundamental impact frequency. For example, a 15 Hz shift in the FIF of the Seta 6 data from 72 Hz - 72.5 Hz results in a 15 Hz shift in the location of the fifth harmonic. Since peaks may be absent at the computed fundamental impact frequency and its harmonics due to this averaging and shift effects, the net result is to give the impression on the spectra of an absence of the fundamental impact frequency and its harmonics. Thus this average and shift effect of power in the computed fundamental impact frequency, due to the rapid variation in the waveform of the impact period, may account for the difficulty encountered by a number of researchers in detecting the computed rotational frequencies in the spectra. This effect then is the common problem encountered in most machines when the main drive "hunts." To insure this effect does not confuse the subsequent analysis, the signal segment length statistics must be studied carefully so that the feature one needs to extract is stationary. The power spectrum gives a measure of this statistic.

6.1 Reasons for the presence of a periodic structure with a Fundamental Periodicity of the Impact Process. An explanation for the presence of a large peak at the bottom race period \( T_B \) can be given by an examination of Fig. 8. The points labeled A and 2 indicate cage periods and it can be seen that the time interval between cage periods is constant over this data segment. This periodicity gives rise to the large peak in the power spectrum at a quasiperiod corresponding to the cage period \( T_B \). Also, it can be seen that within a cage period, the time interval between any four impacts is constant and could correspond to a bottom race defect period. This peak (13) does not indicate a fault and its presence can be explained with respect to the physical events occurring within the Seta mechanism.

The Seta four ball configuration may be considered in terms of an equivalent bearing with an inner and outer race corresponding to the bores of contact as shown in Fig. 9. Compare this with the schematic shown in the Appendix.

Assume that the lower balls are free to spin on their axes but are fixed to the outer race by means of a pin joining the directions of rotation. As the upper ball, UB, rotates, its defect will impact with the inner lower ball and thus the test. The time it takes the upper ball to rotate round the lower balls \( T_B \) will be referred to as the impact cycle (i.e., a normal bearing this is equivalent to the upper race defect). Therefore, in a complete impact cycle the defecive upper ball may generate the impact sequence, A, B, C, or D, E, F etc., seen in Fig. 8.

Although the time between individual impacts may vary (A - B; B - C, etc.), the time between complete impact cycles \( T_B \) is constant (A - B; D - G; A - D, etc.). It is this time which corresponds to the bottom race period, and is responsible for the peak in the power spectrum at a quasiperiod corresponding to three times the impact period.

Hence the apparent cage and bottom race periodicities are produced as a result of the cycle nature of the events within a cage period and are not due to defects in the cage or bottom race.

6.2 Defect Rotationality of the Doublet Structures in Terms of an Intermodulation Model of the Defective Signal. The Seta defective diagnostic signal, Fig. 8, is made up of a periodic sequence of decaying sinusoids and can be decomposed into three constituent parts as described below:

(i) The frequency of summation of a decaying sinusoid, \( f_c \).

This frequency corresponds to one of the resonances of the Seta assembly excited by the impact energy. For the Seta data, the measured resonant frequency was 300 Hz.

(ii) The time varying amplitude of the diagnostic signal with a maximum value of \( \Lambda \) at the impact instant and a minimum value of \( R \) once \( T \) corresponding to the period of the impact sequence. This saw-tooth amplitude profile of the diagnostic signal, called the high frequency envelope structure, carries the information about the Seta state. Consequently, the Seta diagnostic signal can be viewed as the result of an amplitude modulation in which a Seta natural frequency, \( f_c \), acts as the carrier and the upper ball rotational frequency is the modulating frequency.

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(ii) A time varying amplitude profile, called the low frequency envelope structure, can be modeled as an amplitude modulation in which the upper ball rotational frequency is the carrier, and the signal frequency acts as the modulating frequency. Since the modulating signals can be expressed as a Fourier series, the modulated diagnostic signal can be written as:

\[ y(t) = D(1 + \sum_{k=1}^{\infty} B_k \cos(\omega_k t)) \cos(\omega_d t) \]  

where

\[ x(t) = D(1 + \sum_{k=1}^{\infty} D_k \cos(\omega_k t)) \cos(\omega_d t) \]

is the Fourier series representation of the modulating impact signal, with phase information neglected.

The authors are indebted to the Admiralty Maritime Technology Establishment (Holton Heath) for the use of their facilities and in particular for the help given by Mr. D. L. Mack and Dr. J. B. Jones for the ferrographic analysis and many shared discussions.

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APPENDIX

The Seta four-ball machine provides a simple and reliable
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